PRACTICE EXAM 11: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. The expression x^(2/3) · x^(1/4) is equivalent to

A. x^(1/2)

B. x^(3/7)

C. x^(11/12)

D. x^(1/6)

2. The expression $(2 - 3i)^2$, expressed in a + bi form, equals

A. $-5 + 12i$

B. $4 + 9i$

C. $13 - 12i$

D. $-5 - 12i$

3. The expression $27y^3 + 8$, factored completely over the integers, is

A. $(3y + 2)(9y^2 + 6y + 4)$

B. $(3y + 2)(9y^2 - 6y + 4)$

C. $(3y - 2)(9y^2 + 6y + 4)$

D. $(3y - 2)(9y^2 - 6y + 4)$

4. When the polynomial $4x^3 - 3x + 2$ is divided by $(x - 2)$, the remainder is

A. 28

B. -28

C. 14

D. 24

5. When the equation $x^2 - 8x + 6 = 0$ is solved by completing the square, it can be rewritten as

A. $(x + 4)^2 = 10$

B. $(x - 4)^2 = -10$

C. $(x - 4)^2 = 10$

D. $(x + 4)^2 = -6$

6. What is the solution to the equation $5/(x - 4) = (2x)/(x - 4) - 1$?

A. $x = 4$

B. no solution

C. $x = 1$

D. $x = -1$

7. The solution set of the equation $x - \sqrt{(x + 5)} = 1$ is

A. $\{-1, 4\}$

B. $\{4\}$

C. $\{-1\}$

D. $\{\ \}$

8. A polynomial function $p(x)$ has zeros at $x = 1$ (multiplicity 1), $x = -2$ (multiplicity 2), and $x = 5$ (multiplicity 1). Which polynomial, written with leading coefficient 1, satisfies these conditions?

A. $p(x) = (x - 1)(x + 2)^2(x - 5)$

B. $p(x) = (x + 1)(x - 2)^2(x + 5)$

C. $p(x) = (x - 1)^2(x + 2)(x - 5)$

D. $p(x) = (x - 1)(x - 2)^2(x + 5)$

9. For what value of c will the system $y = x^2 + 2x$ and $y = c$ have exactly one real solution?

A. $c = 0$

B. $c = 1$

C. $c = 2$

D. $c = -1$

10. Working alone, Sam can complete a project in 6 hours, and Lee can complete the same project in 8 hours. Which equation can be used to determine the time t (in hours) required to finish the project if they work together?

A. $6t + 8t = 1$

B. $(1/6) + (1/8) = t$

C. $(1/6) + (1/8) = 1/t$

D. $6 + 8 = t$

11. If $f(x) = \sqrt{(x + 4)}$ and $g(x) = x^2 - 4$, then the value of $g(f(5))$ is

A. 9

B. 5

C. −4

D. 25

12. The inverse of the function $f(x) = 2 - 5x$ is $f^{-1}(x)$ equal to

A. $(2 - x) / 5$

B. $(x - 2) / 5$

C. $(x + 2) / 5$

D. $(5 - x) / 2$

13. Solve algebraically for x: 3^(x + 2) = 81.

A. $x = 4$

B. $x = -2$

C. $x = 1$

D. $x = 2$

14. The exact value of $\tan(7\pi/4)$ is

A. 1

B. $\sqrt{2} / 2$

C. -1

D. $-\sqrt{2} / 2$

15. The expression $3 \log_2(x) + \log_2(y) - \log_2(z)$, written as a single logarithm, is

A. $\log_2(x^3y / z)$

B. $\log_2((3x + y) / z)$

C. $3 \log_2(xy / z)$

D. $\log_2(3xy - z)$

16. A sequence is defined recursively by $a_1 = 12$ and $a_n = a_{n-1} - 5$ for $n \geq 2$. The 10th term of the sequence is

A. -27

B. -33

C. -38

D. 33

17. A geometric sequence has first term 4 and common ratio 1/2. The 5th term of the sequence is

A. 1/2

B. 1/4

C. 1/8

D. 2

18. The maximum value of the function $f(x) = -3\sin(2x) + 4$ is

A. -3

B. 3

C. 4

D. 7

19. Solve algebraically for x: $\log_3(x - 4) = 2$.

A. $x = 13$

B. $x = 9$

C. $x = -2$

D. $x = 7$

20. If $\sin\theta = 7/25$ and θ is in Quadrant II, the value of $\cos\theta$ is

A. 24/25

B. 7/24

C. −24/25

D. −7/24

21. A confidence interval for a population proportion is reported as 38% ± 4% at the 95% confidence level. The 95% confidence interval is

A. (36%, 40%)

B. (38%, 42%)

C. (34%, 38%)

D. (34%, 42%)

22. A research team randomly selects 200 students from a list of 5,000 students using a random number generator, giving each student an equal probability of being chosen. This is an example of

A. simple random sampling

B. stratified random sampling

C. cluster sampling design

D. convenience sampling method

23. At a school, 40% of students play a sport, 30% are in a club, and 15% do both. The probability that a randomly selected student plays a sport or is in a club is

A. 0.70

B. 0.55

C. 0.85

D. 0.45

24. A teacher arranges 4 distinct books in a row on a shelf. In how many different orders can the books be arranged?

A. 4

B. 12

C. 16

D. 24

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $4x^2 - 4x + 5 = 0$ algebraically. Express the solutions in a + bi form.

26. Determine the inverse of the function $f(x) = 4 - 2x$ algebraically, and verify your answer by showing that $f(f^{-1}(x)) = x$.

27. Use the Factor Theorem to determine whether $(x - 3)$ is a factor of the polynomial $2x^3 - 7x^2 + 4x + 3$. Justify your conclusion with the appropriate value of the polynomial.

28. Solve the equation e^(2x + 1) = 7 algebraically. Express your answer in exact form using a natural logarithm.

29. Find the sum of the arithmetic series $1 + 4 + 7 + \cdots + 79$. Show all algebraic work using the arithmetic series formula.

30. Solve the equation $2\sin(x) - 1 = 0$ for all values of x in the interval $[0, 2\pi)$. Show how you determined each solution.

31. Use the Pythagorean identity to verify algebraically that $1 - 2\sin^2\theta = 2\cos^2\theta - 1$. Show all algebraic steps.

32. A spinner has 5 equal sectors numbered 1 through 5. The spinner is spun twice and the numbers are recorded. Determine the probability that the product of the two numbers is greater than 12. Express your answer as a fraction in simplest form.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. A company's profit P (in thousands of dollars) is modeled by $P(t) = 50(1.08)^t$, where t is the number of years since the company opened.

(a) State the initial profit (when t = 0) and the annual growth rate as a percentage. Interpret each value in the context of the problem.

(b) Algebraically determine the year in which the profit will reach $100,000 (i.e., 100 thousand dollars). Round your answer to the nearest tenth of a year.

34. The polynomial $p(x) = x^4 - x^3 - 7x^2 + x + 6$ has known real roots $x = 1$ and $x = -2$.

(a) Use synthetic division to divide $p(x)$ by $(x - 1)$. Show your synthetic division work.

(b) Use synthetic division again on the result from part (a) to divide by $(x + 2)$. Show your work.

(c) Factor $p(x)$ completely over the integers, and state all four real solutions to $p(x) = 0$.

35. A pollster randomly surveys 400 voters and finds that 240 support a particular policy.

(a) Determine the sample proportion $\hat{p}$ of voters who support the policy.

(b) Construct a 95% confidence interval for the true population proportion using the formula $ME = 1.96 \cdot \sqrt{(\hat{p}(1 - \hat{p})/n)}$. Round the bounds to three decimal places.

(c) Based on the interval from part (b), can the pollster confidently report that more than half of voters support the policy? Justify your answer.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem integrates concepts from several chapters.

36. The amount of light, L (in lumens), striking a solar panel at time t hours after sunrise on a particular day is modeled by $L(t) = 800 \sin((\pi/12)\, t)$ for $0 \leq t \leq 12$, where $t = 0$ corresponds to sunrise and $t = 12$ corresponds to sunset.

(a) Determine the time at which the solar panel receives the maximum light intensity and state the maximum intensity in lumens. Justify your reasoning based on the model.

(b) Determine the amount of light striking the solar panel at t = 4 hours after sunrise. Round your answer to the nearest lumen.

(c) Algebraically determine all values of t in the interval [0, 12] at which the solar panel receives exactly 400 lumens of light. Round each value of t to the nearest hundredth of an hour. Show all algebraic work.

PRACTICE EXAM 11 – ANSWER KEY AND EXPLANATIONS

1. C — When multiplying powers with the same base, add the exponents: 2/3 + 1/4 = 8/12 + 3/12 = 11/12. The result is x^(11/12), found by converting to a common denominator before summing.

2. D — Apply the FOIL pattern or $(a - b)^2$ formula: $(2 - 3i)^2 = 4 - 12i + 9i^2$. Since $i^2 = -1$: $9i^2 = -9$, giving $4 - 9 - 12i = -5 - 12i$ in standard form.

3. B — Apply the sum of cubes identity $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ with a = 3y and b = 2: $(3y + 2)((3y)^2 - (3y)(2) + 2^2) = (3y + 2)(9y^2 - 6y + 4)$. The middle term takes a negative sign for sums of cubes.

4. A — Apply the Remainder Theorem: dividing by (x − 2) means evaluating at x = 2. Compute $p(2) = 4(8) - 3(2) + 2 = 32 - 6 + 2 = 28$. The polynomial has no x^2 term, but the formula still applies directly.

5. C — Move the constant: $x^2 - 8x = -6$. Add $(8/2)^2 = 16$ to both sides: $x^2 - 8x + 16 = 10$. The left side is a perfect square: $(x - 4)^2 = 10$.

6. C — Restriction $x \neq 4$. Multiply both sides by (x − 4): $5 = 2x - (x - 4)$, giving $5 = x + 4$ and x = 1. Since $1 \neq 4$, the solution is valid; verification: $5/(-3) = -5/3$ and $2(1)/(-3) - 1 = -2/3 - 1 = -5/3$ ✓.

7. B — Isolate the radical: $\sqrt{(x + 5)} = x - 1$. Square both sides: $x + 5 = x^2 - 2x + 1$, giving $x^2 - 3x - 4 = 0$ and $(x - 4)(x + 1) = 0$. Check x = 4: $\sqrt{9} = 3 = 4 - 1$ ✓. Check x = −1: $\sqrt{4} = 2$ but $-1 - 1 = -2$ (extraneous). Solution set: {4}.

8. A — Each zero contributes the factor (x − zero) raised to its multiplicity. Zeros at 1, −2, 5 with multiplicities 1, 2, 1 produce factors (x − 1), $(x + 2)^2$, and (x − 5). Total degree: 1 + 2 + 1 = 4 ✓.

9. D — The parabola $y = x^2 + 2x$ opens upward with vertex at $x = -b/(2a) = -1$, where the minimum value is $(-1)^2 + 2(-1) = -1$. The horizontal line $y = c$ intersects the parabola at exactly one point only when c equals this minimum: $c = -1$.

10. C — In a combined-work problem, individual rates add to give the combined rate. Sam's rate is 1/6 project/hr, Lee's rate is 1/8 project/hr, and the combined rate is 1/t. The equation 1/6 + 1/8 = 1/t encodes this addition.

11. B — Composition is evaluated inside-out. First compute $f(5) = \sqrt{(5 + 4)} = \sqrt{9} = 3$, then $g(3) = 3^2 - 4 = 5$. The output of f becomes the input of g.

12. A — Swap x and y in $y = 2 - 5x$ to get $x = 2 - 5y$. Solve for y: $5y = 2 - x$, so $y = (2 - x)/5$. Verification: $f(f^{-1}(x)) = 2 - 5 \cdot (2 - x)/5 = 2 - (2 - x) = x$ ✓.

13. D — Rewrite 81 as 3^4, giving $3^{(x + 2)} = 3^4$. Equate exponents: $x + 2 = 4$, so $x = 2$. Matching bases reduces an exponential equation to a linear one.

14. C — The angle $7\pi/4$ lies in Quadrant IV with reference angle $2\pi - 7\pi/4 = \pi/4$. Tangent is negative in Quadrant IV (sine is negative, cosine is positive), and $\tan(\pi/4) = 1$, so $\tan(7\pi/4) = -1$.

15. A — Apply the power rule first: $3 \log_2(x) = \log_2(x^3)$. Then apply the product and quotient rules: $\log_2(x^3) + \log_2(y) - \log_2(z) = \log_2(x^3y / z)$. The exponent moves into the argument as a power, while sums and differences become products and quotients.

16. B — The recursion subtracts 5 each step, making this an arithmetic sequence with $a_1 = 12$ and $d = -5$. Apply $a_n = a_1 + (n - 1)d$: $a_{10} = 12 + 9(-5) = 12 - 45 = -33$.

17. B — Apply $a_n = a_1 \cdot r^{(n - 1)}$ with $a_1 = 4$ and $r = 1/2$: $a_5 = 4 \cdot (1/2)^4 = 4 \cdot 1/16 = 1/4$. The exponent is $n - 1$, not n.

18. D — For $y = A \sin(Bx) + D$, the maximum is midline + amplitude = $D + |A|$. Here $D = 4$ and $|A| = 3$, so the maximum is $4 + 3 = 7$. The negative sign on A only affects where the maximum occurs, not its value.

19. A — Convert to exponential form: $3^2 = x - 4$, giving $9 = x - 4$ and $x = 13$. Verification: $\log_3(13 - 4) = \log_3(9) = 2$ ✓.

20. C — Apply the Pythagorean identity: $\cos^2\theta = 1 - \sin^2\theta = 1 - 49/625 = 576/625$, so $|\cos\theta| = 24/25$. In Quadrant II cosine is negative: $\cos\theta = -24/25$.

21. D — The margin of error gives the half-width of the confidence interval: lower bound = 38% − 4% = 34%, upper bound = 38% + 4% = 42%. The interval is (34%, 42%).

22. A — Simple random sampling gives every member of the population an equal and independent chance of being selected. Using a random number generator to draw 200 students from the full roster of 5,000 fits this definition. Stratified, cluster, and convenience methods involve subgroups or non-random selection.

23. B — Apply the addition rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.40 + 0.30 - 0.15 = 0.55$. Subtracting the intersection prevents double-counting students who do both.

24. D — Arrangements of n distinct objects in a row use factorials: $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$. Each position is filled by one of the remaining unplaced books.

PART II — Short Constructed-Response Explanations

25. [2 credits] Apply the quadratic formula with $a = 4$, $b = -4$, $c = 5$: $x = [4 \pm \sqrt{(16 - 80)}]/8 = [4 \pm \sqrt{(-64)}]/8 = [4 \pm 8i]/8 = (1 \pm 2i)/2$. Solutions in a + bi form: **$x = 1/2 + i$ and $x = 1/2 - i$**.

26. [2 credits] Swap x and y in $y = 4 - 2x$ to get $x = 4 - 2y$. Solve for y: $2y = 4 - x$, so **$f^{-1}(x) = (4 - x)/2$**. Verification: $f(f^{-1}(x)) = 4 - 2 \cdot ((4 - x)/2) = 4 - (4 - x) = \mathbf{x}$ ✓.

27. [2 credits] By the Factor Theorem, $(x - 3)$ is a factor if and only if $p(3) = 0$. Compute $p(3) = 2(27) - 7(9) + 4(3) + 3 = 54 - 63 + 12 + 3 = \mathbf{6 \neq 0}$. Since the value is nonzero, **$(x - 3)$ is NOT a factor** of $2x^3 - 7x^2 + 4x + 3$.

28. [2 credits] Take the natural logarithm of both sides: ln(e^(2x + 1)) = ln(7), giving 2x + 1 = ln(7). Solve for x: 2x = ln(7) − 1, so **x = (ln(7) − 1)/2** (exact form).

29. [2 credits] Find n using the last term: $79 = 1 + 3(n - 1)$, so $78 = 3(n - 1)$ and $n = 27$. Apply $S_n = (n/2)(a_1 + a_n)$: $S_{27} = (27/2)(1 + 79) = (27/2)(80) = 27 \cdot 40 =$ **1,080**.

30. [2 credits] Solve $\sin(x) = 1/2$. Reference angle is $\pi/6$ (since $\sin(\pi/6) = 1/2$). Sine is positive in Quadrants I and II. Quadrant I: **$x = \pi/6$**. Quadrant II: $x = \pi - \pi/6 =$ **$5\pi/6$**. Both lie within $[0, 2\pi)$.

31. [2 credits] Apply the Pythagorean identity $\sin^2\theta = 1 - \cos^2\theta$ to rewrite the left side: $1 - 2\sin^2\theta = 1 - 2(1 - \cos^2\theta)$. Distribute: $1 - 2 + 2\cos^2\theta$. Combine constants: **$2\cos^2\theta - 1$** ✓. Both sides agree, so the identity is verified.

32. [2 credits] Total outcomes for two spins: $5 \times 5 = 25$. Count outcomes (a, b) where $a \cdot b > 12$: (3, 5) = 15, (4, 4) = 16, (4, 5) = 20, (5, 3) = 15, (5, 4) = 20, (5, 5) = 25 — six favorable outcomes. Therefore P(product > 12) = **6/25**.

PART III — Extended Constructed-Response Explanations

33.

(a) [2 credits] Initial profit: $P(0) = 50(1.08)^0 =$ **50 thousand dollars ($50,000)** — the profit at the moment the company opened. Growth rate: since the growth factor is 1.08 = 1 + 0.08, the annual growth rate is **8%** — the company's profit increases by 8% each year.

(b) [2 credits] Set $P(t) = 100$: $50(1.08)^t = 100$, so $(1.08)^t = 2$. Take the natural logarithm: $t \cdot \ln(1.08) = \ln(2)$, giving $t = \ln(2)/\ln(1.08) \approx 0.6931/0.07696 \approx$ **9.0 years**.

34.

(a) [1 credit] Synthetic division with k = 1, coefficients 1, −1, −7, 1, 6:

```
1 |  1   −1   −7    1    6
  |       1    0   −7   −6
  |_____________________________
    1    0   −7   −6    0
```

Quotient: **$x^3 - 7x - 6$**.

(b) [1 credit] Synthetic division with k = −2, coefficients 1, 0, −7, −6:

```
−2 |  1    0   −7   −6
   |      −2    4    6
   |_____________________
     1   −2   −3    0
```

Quotient: **$x^2 - 2x - 3$**.

(c) [2 credits] Factor: $x^2 - 2x - 3 = (x - 3)(x + 1)$. Complete factorization: $p(x) =$ **$(x - 1)(x + 2)(x - 3)(x + 1)$**. Real solutions: **$x = 1, x = -2, x = 3,$ and $x = -1$**.

35.

(a) [1 credit] Sample proportion: $\hat{p} = 240/400 =$ **0.6** (or 60%).

(b) [2 credits] Compute $ME = 1.96 \cdot \sqrt{(0.6 \cdot 0.4/400)} = 1.96 \cdot \sqrt{(0.24/400)} = 1.96 \cdot \sqrt{0.0006} \approx 1.96 \cdot 0.02449 \approx 0.04801$. Confidence interval: 0.6 ± 0.04801 — lower bound $0.6 - 0.04801 \approx$ **0.552**, upper bound $0.6 + 0.04801 \approx$ **0.648**. CI ≈ **(0.552, 0.648)**.

(c) [1 credit] **Yes** — the entire interval (0.552, 0.648) lies above 0.50, and the lower bound 0.552 is greater than 0.50. The pollster can confidently report at the 95% confidence level that more than half of voters support the policy, since the data are inconsistent with the true proportion being 0.50 or less.

PART IV — Long Constructed-Response Explanation

36.

(a) [2 credits] The sine function reaches its maximum value of 1 when its argument equals $\pi/2$. Setting $(\pi/12)\,t = \pi/2$ gives $t - 4 = 6$, so **t = 6 hours after sunrise** (midday). The maximum intensity is $L(6) = 800 \cdot \sin(\pi/2) = 800 \cdot 1 =$ **800 lumens**. This corresponds to the sun being directly overhead.

(b) [1 credit] Substitute t = 4: $L(4) = 800 \sin((\pi/12)(4)) = 800 \sin(\pi/3)$. Since $\sin(\pi/3) = \sqrt{3}/2$: $L(4) = 800 \cdot \sqrt{3}/2 = 400\sqrt{3} \approx 692.82$, rounded to **693 lumens**.

(c) [3 credits] Set L(t) = 400: 800 sin((π/12) t) = 400, so sin((π/12) t) = 1/2. Let u = (π/12) t; as t ranges over [0, 12], u ranges over [0, π]. In [0, π], sin(u) = 1/2 has solutions $u_1 = \pi/6$ and $u_2 = 5\pi/6$. Convert back using t = (12/π) u: $t_1 = (12/\pi)(\pi/6)$ = **2.00 hours** and $t_2 = (12/\pi)(5\pi/6)$ = **10.00 hours**. The solar panel receives exactly 400 lumens at 2 hours after sunrise (mid-morning) and 10 hours after sunrise (mid-afternoon), symmetrically placed around the noon maximum at t = 6.