PRACTICE EXAM 10: ALGEBRA II REGENTS SIMULATION

PART I — Multiple Choice (24 questions × 2 credits = 48 credits)

1. The expression $(16x^8)^{(3/4)}$ is equivalent to

A. $12x^6$

B. $8x^6$

C. $8x^{14}$

D. $16x^6$

2. The product $(1 + 4i)(3 - 2i)$, expressed in $a + bi$ form, is

A. $11 - 10i$

B. $3 - 8i$

C. $11 + 10i$

D. $-5 + 10i$

3. A polynomial has roots $x = 2$, $x = -3$ (multiplicity 2), and $x = 5$. What is the minimum degree of the polynomial?

A. 4

B. 3

C. 5

D. 6

4. When the polynomial $3x^3 + 2x^2 - 5x + 1$ is divided by $(x + 1)$, the remainder is

A. 1

B. -5

C. 3

D. 5

5. The roots of $x^2 - 6x + 4 = 0$ are

A. $3 \pm \sqrt{5}$

B. $-3 \pm \sqrt{5}$

C. $3 \pm 2\sqrt{5}$

D. $3 \pm \sqrt{20}$

6. What is the solution to the equation $3/x + 1/(x + 2) = 1$?

A. $1 \pm \sqrt{7}$

B. $1 \pm 2\sqrt{7}$

C. $-1 \pm \sqrt{7}$

D. $2 \pm \sqrt{7}$

7. What is the solution set of $\sqrt{(2x + 1)} = x - 1$?

A. {0, 4}

B. {0}

C. {4}

D. { }

8. Which ordered pair is a solution to the system $y = -x^2 + 3$ and $y = x + 1$?

A. (−2, 3)

B. (1, 0)

C. (2, 3)

D. (1, 2)

9. A ball thrown into the air has its height (in meters) modeled by $h(t) = -5(t - 2)^2 + 25$, where t is in seconds. The maximum height the ball reaches and the time at which it occurs are

A. Maximum 20 m at t = 2 seconds

B. Maximum 25 m at t = 2 seconds

C. Maximum 25 m at t = 5 seconds

D. Maximum 5 m at t = 25 seconds

10. The expression $4x^4 - 25y^2$, factored completely over the integers, is

A. $(2x^2 - 5y)(2x^2 + 5y)$

B. $(2x - 5y)(2x + 5y)$

C. $(2x^2 - 5y)^2$

D. $(4x^2 - 5y)(x^2 + 5y)$

11. If $f(x) = x^2 + 2x - 3$, then the value of $f(-3)$ is

A. -6

B. 3

C. 0

D. 6

12. A geometric sequence has first term $a_1 = 5$ and third term $a_3 = 45$. If $r > 0$, then the common ratio r is

A. 9

B. 5

C. 8

D. 3

13. The function $f(x)$ is transformed to produce $g(x) = 2f(x - 3)$. The graph of $g(x)$ compared to $f(x)$ is shifted

A. left 3 units and vertically stretched by a factor of 2

B. right 3 units and vertically stretched by a factor of 2

C. left 3 units and vertically compressed by a factor of 1/2

D. right 3 units and vertically compressed by a factor of 1/2

14. Solve algebraically: $4^{(x + 1)} = 8^{(x - 1)}$.

A. $x = 3$

B. $x = 2$

C. $x = 5$

D. $x = -1$

15. The exact value of $\sin(4\pi/3)$ is

A. $-1/2$

B. $1/2$

C. $\sqrt{3}/2$

D. $-\sqrt{3}/2$

16. What is the solution to the equation $\log_4(x + 3) = 2$?

A. $x = 5$

B. $x = -1$

C. $x = 8$

D. $x = 13$

17. The sum of the first 25 terms of the arithmetic sequence 2, 5, 8, 11, … is

A. 750

B. 800

C. 950

D. 1,250

18. The inverse of the function $f(x) = \sqrt[3]{x} - 2$ is $f^{-1}(x)$ equal to

A. $(x + 2)^3$

B. $\sqrt[3]{(x + 2)}$

C. $(x - 2)^3$

D. $x^3 + 2$

19. Which sinusoidal function has amplitude 3, period 2π, midline $y = 1$, and a maximum value at $x = 0$?

A. $y = 3 \sin(x) + 1$

B. $y = \sin(3x) + 1$

C. $y = \cos(3x) + 1$

D. $y = 3 \cos(x) + 1$

20. The expression (1/2) log(x) + 2 log(y) − log(z), written as a single logarithm, is

A. $\log(xy^2 / z)$

B. $\log(y^2 \sqrt{x} / z)$

C. $\log((x + 2y) / z)$

D. $\log((x/2) (y^2) / z)$

21. On a standardized test, a student scored at the 84th percentile. The correct interpretation of this score is that

A. the student answered 84% of the test questions correctly

B. the student got exactly 84 questions correct on the test

C. the student scored higher than 84% of those who took the test

D. the student is in the top 84% of all students who took the test

22. A normal distribution has a mean of 50 and a standard deviation of 10. Approximately what percent of values fall between 30 and 70?

A. 95%

B. 68%

C. 50%

D. 99.7%

23. A fair coin is flipped three times. What is the probability of obtaining heads on all three flips?

A. 1/2

B. 1/8

C. 3/8

D. 1/3

24. In a class of 30 students, 20 students like math, 15 students like science, and 10 students like both subjects. If a randomly selected student likes math, what is the probability that the student also likes science?

A. 10/30

B. 15/30

C. 10/15

D. 10/20

PART II — Short Constructed Response (8 questions × 2 credits = 16 credits)

Show all work. A correct answer with no supporting work will receive only 1 credit.

25. Solve the equation $5x^2 + 6x + 5 = 0$ algebraically. Express the solutions in $a + bi$ form.

26. Simplify the rational expression $(x^2 - x - 12) / (x^2 - 9)$ for $x \neq \pm 3$. Show all factoring steps.

27. A polynomial $p(x)$ with leading coefficient 2 has linear factors $(x - 1)$, $(x + 2)$, and $(x - 5)$. Write $p(x)$ as a polynomial in standard form in expanded form.

28. Solve the logarithmic equation $\ln(2x) = 3$ algebraically. Express your answer in exact form using e.

29. A geometric sequence has nth term $a_n = 5 \cdot 2^{(n-1)}$. Find the sum of the first 8 terms of the sequence. Show all algebraic work using the geometric series formula.

30. Verify the trigonometric identity $\tan^2\theta + 1 = \sec^2\theta$ algebraically. Show every step using Pythagorean identities and the definitions of tan and sec.

31. Solve the equation $2 \sin(x) \cos(x) = 1$ for all values of x in the interval $[0, 2\pi)$. Show all algebraic work using a double-angle identity.

32. A jar contains 5 red marbles, 4 blue marbles, and 3 green marbles. Three marbles are drawn at random without replacement. Determine the probability that all three marbles drawn are red. Express your answer as a fraction in simplest form.

PART III — Extended Constructed Response (3 questions × 4 credits = 12 credits)

Show all work. Partial credit is awarded according to the scoring rubric.

33. A bacterial culture begins with 500 cells and grows according to the function $P(t) = 500\, e^{(0.2t)}$, where t is measured in hours.

(a) Determine the bacterial population after 8 hours. Round your answer to the nearest whole number.

(b) Algebraically determine the time at which the population reaches 5,000 cells. Round your answer to the nearest tenth of an hour.

34. The polynomial function $f(x) = 2x^3 + 3x^2 - 11x - 6$ has $x = 2$ as a known real root.

(a) Use synthetic division to find the quotient when $f(x)$ is divided by $(x - 2)$. Show your synthetic division work.

(b) Factor $f(x)$ completely over the integers.

(c) State all real solutions to the equation $f(x) = 0$.

35. A streaming service collects sample proportions of customers who hold premium subscriptions across four years. The randomized sample of 200 customers in each year yields these proportions: Year 1: 0.30; Year 2: 0.35; Year 3: 0.42; Year 4: 0.48.

(a) Determine the mean proportion $\bar{p}$ of premium subscribers across the four years. Round to three decimal places.

(b) Compute a 95% confidence interval for the population proportion of premium subscribers in Year 4, using the formula $ME = 1.96 \cdot \sqrt{\hat{p}(1 - \hat{p})/n}$ with $\hat{p} = 0.48$ and $n = 200$. Round the bounds to three decimal places.

(c) Based on the interval from part (b), is there strong evidence that the true Year 4 proportion exceeds 0.40? Justify your answer.

PART IV — Long Constructed Response (1 question × 6 credits = 6 credits)

Show all work. This problem integrates concepts from several chapters.

36. A Ferris wheel has a diameter of 30 meters, with the center of the wheel located 18 meters above the ground. The wheel rotates at a steady rate, completing one full revolution every 4 minutes. At time $t = 0$ minutes, a rider boards at the lowest point of the wheel.

(a) Write a sinusoidal function $H(t)$ that gives the height (in meters) of the rider above the ground at time t (in minutes). State and justify your choice of trigonometric function, amplitude, period, midline, and vertical shift.

(b) Determine the rider's height at $t = 1.5$ minutes. Round your answer to the nearest tenth of a meter.

(c) Algebraically determine all values of t in the interval [0, 8] minutes at which the rider is exactly 25 meters above the ground. Round each value of t to the nearest hundredth of a minute. Show all algebraic work.

PRACTICE EXAM 10 – ANSWER KEY AND EXPLANATIONS

1. B — Apply the rational exponent rule: $(16x^8)^{(3/4)} = 16^{(3/4)} \cdot x^{(8 \cdot 3/4)}$. Since $16 = 2^4$, the first factor is $(2^4)^{(3/4)} = 2^3 = 8$, and the second factor is x^6. Result: $8x^6$.

2. C — Distribute (FOIL): $(1 + 4i)(3 - 2i) = 3 - 2i + 12i - 8i^2$. Since $i^2 = -1$: $-8i^2 = +8$, so the expression becomes $3 + 8 + 10i = 11 + 10i$ in standard form.

3. A — The minimum degree of a polynomial equals the sum of all multiplicities of its roots. With roots of multiplicity 1, 2, and 1, the minimum degree is $1 + 2 + 1 = 4$.

4. D — Apply the Remainder Theorem: dividing by $(x + 1)$ means evaluating at $x = -1$. Compute $p(-1) = 3(-1)^3 + 2(-1)^2 - 5(-1) + 1 = -3 + 2 + 5 + 1 = 5$.

5. A — Apply the quadratic formula with $a = 1$, $b = -6$, $c = 4$: $x = [6 \pm \sqrt{(36 - 16)}]/2 = [6 \pm \sqrt{20}]/2 = [6 \pm 2\sqrt{5}]/2 = 3 \pm \sqrt{5}$. Simplifying $\sqrt{20}$ to $2\sqrt{5}$ is essential for the simplest radical form.

6. A — The LCD is $x(x + 2)$. Multiply both sides: $3(x + 2) + x = x(x + 2)$, giving $4x + 6 = x^2 + 2x$ and $x^2 - 2x - 6 = 0$. Quadratic formula: $x = [2 \pm \sqrt{(4 + 24)}]/2 = [2 \pm 2\sqrt{7}]/2 = 1 \pm \sqrt{7}$. Both values satisfy $x \neq 0$ and $x \neq -2$.

7. C — Square both sides: $2x + 1 = x^2 - 2x + 1$, giving $x^2 - 4x = 0$ and $x(x - 4) = 0$. Check $x = 0$: $\sqrt{1} = 1 \neq -1$ (extraneous, principal root cannot be negative). Check $x = 4$: $\sqrt{9} = 3 = 4 - 1$ ✓. Solution set: {4}.

8. D — Substitute to eliminate y: $x + 1 = -x^2 + 3$, giving $x^2 + x - 2 = 0$ and $(x + 2)(x - 1) = 0$. At $x = 1$: $y = 1 + 1 = 2$, yielding (1, 2), which appears among the options. The point (−2, −1) is the other intersection but is not listed.

9. B — In vertex form $h(t) = a(t - h)^2 + k$, the vertex is (h, k). With $-5(t - 2)^2 + 25$, the vertex is (2, 25). The negative leading coefficient makes the parabola open downward, so the vertex represents a maximum.

10. A — Recognize $4x^4$ as $(2x^2)^2$ and $25y^2$ as $(5y)^2$. Apply the difference of squares: $a^2 - b^2 = (a - b)(a + b)$, giving $(2x^2 - 5y)(2x^2 + 5y)$. The factor $(2x^2 - 5y)$ does not factor further over the integers.

11. C — Substitute $x = -3$: $f(-3) = (-3)^2 + 2(-3) - 3 = 9 - 6 - 3 = 0$. The value −3 is a root of the polynomial.

12. D — Apply $a_n = a_1 \cdot r^{(n - 1)}$: $a_3 = 5r^2 = 45$, so $r^2 = 9$ and $r = 3$ (taking the positive root since $r > 0$).

13. B — In $g(x) = 2f(x - 3)$, the "−3" inside the function argument shifts the graph horizontally to the right by 3 units (subtraction inside ⇒ rightward shift). The coefficient 2 outside multiplies the output, producing a vertical stretch by factor 2.

14. C — Rewrite both sides with base 2: $4 = 2^2$ and $8 = 2^3$, so $2^{(2(x + 1))} = 2^{(3(x - 1))}$. Equate exponents: $2x + 2 = 3x - 3$, giving $x = 5$.

15. D — The angle $4\pi/3$ lies in Quadrant III with reference angle $\pi/3$. Sine is negative in Quadrant III, and $\sin(\pi/3) = \sqrt{3}/2$, so $\sin(4\pi/3) = -\sqrt{3}/2$.

16. D — Convert to exponential form: $\log_4(x + 3) = 2$ means $4^2 = x + 3$, so $16 = x + 3$ and $x = 13$. Verification: $\log_4(16) = 2$ ✓.

17. C — Apply $S_n = (n/2)(2a_1 + (n - 1)d)$ with $a_1 = 2$, $d = 3$, $n = 25$: $S_{25} = (25/2)(2 \cdot 2 + 24 \cdot 3) = (25/2)(4 + 72) = (25/2)(76) = 25 \cdot 38 = 950$.

18. A — Swap x and y in $y = \sqrt[3]{x} - 2$ to get $x = \sqrt[3]{y} - 2$. Add 2: $x + 2 = \sqrt[3]{y}$. Cube both sides: $y = (x + 2)^3$. Verification: $f(f^{-1}(x)) = \sqrt[3]{((x + 2)^3)} - 2 = (x + 2) - 2 = x$ ✓.

19. D — For amplitude 3 and midline $y = 1$, the form is $y = 3\ \mathrm{trig}(Bx) + 1$. Period 2π means $B = 1$. A maximum at $x = 0$ requires cosine (since $\cos(0) = 1$, giving the maximum), not sine (sine equals 0 at the origin). Hence $y = 3\cos(x) + 1$.

20. B — Apply the power rule: $(1/2)\log(x) = \log(x^{(1/2)}) = \log(\sqrt{x})$, and $2\log(y) = \log(y^2)$. Combine using product and quotient rules: $\log(\sqrt{x}) + \log(y^2) - \log(z) = \log(y^2 \sqrt{x} / z)$.

21. C — The nth percentile is the value at or above which n% of the data fall. Scoring at the 84th percentile means scoring higher than 84% of test-takers. Option A confuses percentile with percent of items correct; option D reverses the direction.

22. A — $30 = 50 - 2(10)$ and $70 = 50 + 2(10)$, so the range is mean ± 2 standard deviations. By the Empirical Rule, approximately 95% of normally distributed data falls within 2 standard deviations of the mean.

23. B — Independent events allow multiplying probabilities. Each flip has P(heads) = 1/2, so P(HHH) = $(1/2) \cdot (1/2) \cdot (1/2) = 1/8$. Independence means earlier results don't change later probabilities.

24. D — Conditional probability restricts to the condition's sample space. Given the student likes math (20 students), the probability they also like science is the count who like both (10) divided by the math count (20): 10/20 = 1/2. Without simplification, the option 10/20 directly matches.

PART II — Short Constructed-Response Explanations

25. [2 credits] Apply the quadratic formula with a = 5, b = 6, c = 5: $x = [-6 \pm \sqrt{36 - 100}]/10 = [-6 \pm \sqrt{-64}]/10 = [-6 \pm 8i]/10 = (-3 \pm 4i)/5$. Solutions in a + bi form: **$x = -3/5 + (4/5)i$ and $x = -3/5 - (4/5)i$**.

26. [2 credits] Factor: $x^2 - x - 12 = (x - 4)(x + 3)$, and $x^2 - 9 = (x - 3)(x + 3)$. Cancel the common factor $(x + 3)$, leaving **$(x - 4)/(x - 3)$** for $x \neq \pm 3$.

27. [2 credits] Compute $(x - 1)(x + 2) = x^2 + x - 2$. Then multiply by $(x - 5)$: $(x^2 + x - 2)(x - 5) = x^3 - 5x^2 + x^2 - 5x - 2x + 10 = x^3 - 4x^2 - 7x + 10$. Apply leading coefficient 2: **$p(x) = 2x^3 - 8x^2 - 14x + 20$**.

28. [2 credits] Convert the natural log to exponential form: $\ln(2x) = 3$ means $e^3 = 2x$. Solve for x: **$x = e^3/2$** (exact form, approximately 10.04).

29. [2 credits] The sequence is geometric with $a_1 = 5$ and $r = 2$. Apply $S_n = a_1(1 - r^n)/(1 - r)$: $S_8 = 5(1 - 2^8)/(1 - 2) = 5(1 - 256)/(-1) = 5(-255)/(-1) =$ **1,275**.

30. [2 credits] Begin with the left side and convert to sines and cosines: $\tan^2\theta + 1 = \sin^2\theta/\cos^2\theta + 1$. Use a common denominator: $\sin^2\theta/\cos^2\theta + \cos^2\theta/\cos^2\theta = (\sin^2\theta + \cos^2\theta)/\cos^2\theta$. Apply the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$: $1/\cos^2\theta =$ **$\sec^2\theta$** ✓. The identity is verified.

31. [2 credits] Apply the double-angle identity $2 \sin(x) \cos(x) = \sin(2x)$, giving $\sin(2x) = 1$. So $2x = \pi/2 + 2\pi k$ for integers k, meaning $x = \pi/4 + \pi k$. In $[0, 2\pi)$: k = 0 gives **$x = \pi/4$**, and k = 1 gives **$x = 5\pi/4$**.

32. [2 credits] Total marbles = 5 + 4 + 3 = 12. Sequential probabilities without replacement: P(all 3 red) = (5/12)(4/11)(3/10) = 60/1,320. Simplify by dividing by GCD 60: **1/22**.

PART III — Extended Constructed-Response Explanations

33.

(a) [2 credits] Substitute t = 8: $P(8) = 500\, e^{(0.2 \cdot 8)} = 500\, e^{(1.6)} \approx 500 \cdot 4.9530 \approx 2{,}476.5$. Rounded to the nearest whole number: **2,477 cells**.

(b) [2 credits] Set $P(t) = 5{,}000$: $500 e^{0.2t} = 5{,}000$, so $e^{0.2t} = 10$. Take the natural logarithm: $0.2t = \ln(10) \approx 2.3026$, giving $t \approx 2.3026/0.2 \approx$ **11.5 hours**.

34.

(a) [2 credits] Apply synthetic division with k = 2 on coefficients 2, 3, −11, −6:

```
2 |  2    3   -11   -6
  |       4    14    6
  |___________________________
     2    7     3    0
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The quotient is **$2x^2 + 7x + 3$** with remainder 0, confirming x = 2 as a root.

(b) [1 credit] Factor $2x^2 + 7x + 3$: split the middle term 7x as 6x + x and group: $2x(x + 3) + 1(x + 3) = (2x + 1)(x + 3)$. Complete factorization: $f(x) =$ **$(x - 2)(2x + 1)(x + 3)$**.

(c) [1 credit] Setting each factor to zero: **x = 2, x = −1/2, and x = −3**.

35.

(a) [1 credit] Mean proportion: $\bar{p} = (0.30 + 0.35 + 0.42 + 0.48)/4 = 1.55/4 =$ **0.388** (rounded to three decimal places).

(b) [2 credits] Compute $ME = 1.96 \cdot \sqrt{0.48 \cdot 0.52/200} = 1.96 \cdot \sqrt{0.2496/200} = 1.96 \cdot \sqrt{0.001248} \approx 1.96 \cdot 0.03533 \approx 0.0692$. Confidence interval: 0.48 ± 0.0692, giving lower bound $0.48 - 0.0692 \approx$ **0.411** and upper bound $0.48 + 0.0692 \approx$ **0.549**. CI ≈ **(0.411, 0.549)**.

(c) [1 credit] The interval (0.411, 0.549) lies **entirely above 0.40** — the lower bound (0.411) is greater than 0.40. **Yes**, there is strong evidence that the true Year 4 proportion exceeds 0.40, because the 95% confidence interval does not include any values at or below 0.40.

PART IV — Long Constructed-Response Explanation

36.

(a) [3 credits] The radius is 30/2 = 15 m, so the amplitude is **15 m**. The center is 18 m above the ground, so the midline is **y = 18 m**. The period is 4 minutes, giving $B = 2\pi/4 = \pi/2$. The rider starts at the lowest point (3 m, since 18 − 15 = 3), which is the minimum of the function; using negative cosine starts at the minimum at t = 0 (because $-\cos(0) = -1$). The model is **$H(t) = -15 \cos((\pi/2)\, t) + 18$**. Verification: $H(0) = -15(1) + 18 = 3$ ✓ (lowest point) and $H(2) = -15(-1) + 18 = 33$ ✓ (highest point after one half-revolution).

(b) [1 credit] Substitute $t = 1.5$: $H(1.5) = -15\cos((\pi/2)(1.5)) + 18 = -15\cos(3\pi/4) + 18$. Since $\cos(3\pi/4) = -\sqrt{2}/2 \approx -0.7071$: $H(1.5) = -15(-0.7071) + 18 \approx 10.607 + 18 \approx$ **28.6 meters**.

(c) [2 credits] Set $H(t) = 25$: $-15\cos((\pi/2)t) + 18 = 25$, giving $-15\cos((\pi/2)t) = 7$ and $\cos((\pi/2)t) = -7/15$. Let $u = (\pi/2)t$; as t ranges over $[0, 8]$, u ranges over $[0, 4\pi]$. The equation $\cos(u) = -7/15$ has solutions $u_1 = \arccos(-7/15) \approx 2.0570$ rad and $u_2 = 2\pi - u_1 \approx 4.2262$ rad in $[0, 2\pi)$. Adding the period 2π gives two more solutions $u_3 \approx 8.3402$ rad and $u_4 \approx 10.5094$ rad in $[2\pi, 4\pi]$. Convert via $t = (2/\pi)u$: $t_1 \approx (2/\pi)(2.0570) \approx$ **1.31 minutes**, $t_2 \approx (2/\pi)(4.2262) \approx$ **2.69 minutes**, $t_3 \approx (2/\pi)(8.3402) \approx$ **5.31 minutes**, $t_4 \approx (2/\pi)(10.5094) \approx$ **6.69 minutes**. The rider is exactly 25 m above the ground four times during the 8-minute window — twice per revolution, symmetrically placed about each maximum at $t = 2$ and $t = 6$.