PRACTICE EXAM 40

NY REGENTS ALGEBRA I SIMULATION — 35 QUESTIONS

Recommended Time: 3 Hours

Required Tools: Graphing Calculator, Straightedge

Directions: Answer all 35 questions. For Part I, record answers on your answer sheet. For Parts II, III, and IV, show all work in the space provided. Partial credit is available on Parts II–IV.

PART I — Multiple Choice (Questions 1–24)

Each correct answer is worth 2 credits. No partial credit. No penalty for guessing.

1. A population of bacteria starts at 200 cells and doubles every hour. Which function models the population P after h hours?

A. P(h) = 200 + 2h

B. P(h) = 200(0.5)^h

C. P(h) = 200(2)^h

D. P(h) = 2(200)^h

2. What is the solution to the equation $5(x - 2) = 2x + 8$?

A. $x = 6$

B. $x = 2$

C. $x = 18$

D. $x = -6$

3. A line passes through the points $(-3, 2)$ and $(3, -6)$. What is the slope of the line?

A. $4/3$

B. $-3/4$

C. $3/4$

D. $-4/3$

4. Which expression is the completely factored form of $x^2 + 3x - 28$?

A. $(x - 7)(x + 4)$

B. $(x + 7)(x - 4)$

C. $(x + 14)(x - 2)$

D. $(x - 14)(x + 2)$

5. If $f(x) = -x^2 + 4x - 5$, what is the value of $f(3)$?

A. $f(3) = -20$

B. $f(3) = 16$

C. $f(3) = 22$

D. $f(3) = -2$

6. What are the solutions to the equation $x^2 - 6x = -8$?

A. $x = -2$ and $x = -4$

B. $x = 1$ and $x = 8$

C. $x = 2$ and $x = 4$

D. $x = -2$ and $x = 4$

7. An arithmetic sequence begins 5, 1, −3, −7, … Which explicit formula correctly represents the nth term?

A. $a_n = 5 - 4(n - 1)$

B. $a_n = 5 + 4(n - 1)$

C. $a_n = -4 + 5(n - 1)$

D. $a_n = 5(-4)^{(n - 1)}$

8. Which inequality represents all values of x for which $3 - 4x > 19$?

A. $x > -4$

B. $x < -4$

C. $x > 4$

D. $x < 4$

9. What is the solution to the system $y = 2x - 1$ and $y = -x + 5$?

A. (2, 3)

B. (3, 2)

C. (2, −3)

D. $(-2, 3)$

10. The function $V(t) = 8000(0.92)^t$ models the value of a car t years after purchase. What is the annual rate of change?

A. 92% increase per year

B. 8% increase per year

C. 92% decrease per year

D. 8% decrease per year

11. Which expression is equivalent to $(3x^2 + 5x - 2) - (x^2 - 2x + 6)$?

A. $2x^2 + 3x + 4$

B. $2x^2 + 7x - 8$

C. $2x^2 + 7x + 4$

D. $4x^2 + 3x - 8$

12. Which statement best describes the transformation from $f(x) = x^2$ to $g(x) = x^2 + 5$?

A. The graph of f(x) is shifted horizontally right 5 units

B. The graph of $f(x)$ is shifted horizontally left 5 units

C. The graph of $f(x)$ is shifted vertically up 5 units

D. The graph of $f(x)$ is shifted vertically down 5 units

13. A line has a slope of 2/3 and a y-intercept of −4. Which equation represents this line written in standard form?

A. $2x - 3y = 12$

B. $2x + 3y = 12$

C. $2x - 3y = -12$

D. $3x - 2y = 12$

14. The function $f(x) = x^2 + 6x + 5$ has a minimum value. What is that minimum value?

A. $f = -3$

B. $f = 5$

C. $f = 9$

D. $f = -4$

15. Which statement correctly classifies the number $\sqrt{20}$?

A. It is rational because it simplifies to $2\sqrt{5}$

B. It is irrational because 20 is not a perfect square

C. It is rational because all square roots are rational

D. It is irrational because it contains a decimal point

16. Consider the data set: 14, 16, 15, 17, 16, 92, 15. Which measure of center best represents a typical value, and why?

A. Mean; the data set has a small overall spread of values

B. Mean; it accounts for every value recorded in the set

C. Median; the value 92 is an outlier that distorts the mean

D. Median; the median is always the best measure of center

17. A gym charges a \$25 sign-up fee plus \$15 per month. Which inequality represents the number of months m a member can belong while keeping the total cost at or below \$160?

A. $m \leq 12$

B. $m \geq 9$

C. $m \leq 135$

D. $m \leq 9$

18. The table below shows values of a function.

x: 0, 1, 2, 3, 4

y: 3, 6, 12, 24, 48

Which type of function is represented?

A. Exponential, with a common ratio of 2

B. Linear, with a constant rate of change of 3

C. Quadratic, with constant second differences

D. Exponential, with a common ratio of 3

19. A study finds that as the number of hours spent studying increases, exam scores increase, with the data points falling very close to an upward-sloping line. Which value is closest to the correlation coefficient?

A. $r = -0.93$

B. $r = 0.10$

C. $r = 0.93$

D. $r = 0.50$

20. Which system of equations has no solution?

A. $y = 4x + 2$ and $y = -4x + 2$

B. $y = 2x + 1$ and $y = 2x - 5$

C. $y = x + 3$ and $y = -x + 3$

D. $y = 3x - 2$ and $y = 5x - 2$

21. Which point lies on the line $y = (1/2)x + 3$?

A. (2, 3)

B. (4, 3)

C. (0, 1/2)

D. (−4, 1)

22. Which expression is the completely factored form of $2x^2 - 18$?

A. $2(x - 3)(x + 3)$

B. $2(x - 3)^2$

C. $(2x - 9)(x + 2)$

D. $2(x - 9)(x + 1)$

23. A data set has the following five-number summary: minimum 4, Q1 = 10, median 14, Q3 = 22, maximum 30. What is the interquartile range, and what does it represent?

A. IQR = 26; the total spread of the entire data set

B. IQR = 8; the distance from the median to the third quartile

C. IQR = 12; the spread of the middle 50% of the data values

D. IQR = 4; the spread of the lower quarter of the data values

24. What are the zeros of the function $f(x) = (2x - 6)(x + 5)$?

A. $x = 6$ and $x = -5$

B. $x = 3$ and $x = -5$

C. $x = -3$ and $x = 5$

D. $x = 6$ and $x = 5$

PART II — Short Constructed Response (Questions 25–32)

Each question is worth 2 credits. Show all work.

25. Solve the following system algebraically and verify your solution.

$3x + 2y = 4$

$x - y = 7$

26. Write the equation of the line that is perpendicular to $y = (1/3)x - 2$ and passes through the point $(3, -5)$. Express your answer in slope-intercept form.

27. A quadratic function has zeros at $x = -5$ and $x = 1$, and a leading coefficient of 2. Write the function in factored form and expand it to standard form.

28. The table below shows the height h (in feet) of a launched object at time t (in seconds).

t (seconds): 0, 1, 2, 3, 4

h(t) (feet): 6, 21, 26, 21, 6

Determine whether the function is linear or quadratic. Justify your answer using the table. Then identify the maximum height and when it occurs.

29. Factor the following expression completely: $4x^3 - 36x$.

30. An arithmetic sequence has $a_3 = 12$ and $a_7 = 32$. Find the common difference, the first term, and write the explicit formula for the nth term.

31. Graph the solution set of the system of inequalities on a coordinate plane. State one point in the solution region and verify it satisfies both inequalities.

$y < (1/2)x + 4$

$y \geq -x - 1$

32. A marketing analyst models weekly sales with a line of best fit $\hat{y} = 2.5x + 30$, where x represents the number of advertisements run and $\hat{y}$ is the predicted number of units sold. In a week with 8 advertisements, the measured number of units sold was 60.

Calculate the residual for that week and explain what the sign of the residual tells you about the data point relative to the line.

PART III — Medium Constructed Response (Questions 33–34)

Each question is worth 4 credits. Show all work.

33. Consider the following assertion:

"For the function $f(x) = x^2 - 10x + 21$, the minimum value of f(x) is −4."

a. Determine whether this assertion is true or false. Show your work by finding the vertex of the function using the axis of symmetry formula.

b. Identify the zeros of f(x) algebraically.

c. Describe the key features of the graph: direction of opening, vertex, axis of symmetry, x-intercepts, and y-intercept. (You do not need to draw the graph — describe each feature precisely.)

34. A tutor records the number of hours each student studied x and that student's exam score y. After collecting data from many students, the line of best fit is determined to be $y = 4x + 52$, with a correlation coefficient of $r = 0.91$.

a. Interpret the slope of the line of best fit in context.

b. Predict the exam score of a student who studies for 9 hours. Show your calculation.

c. A student who studied for 7 hours actually scored 85. Calculate the residual and determine whether the actual score was above or below the predicted value.

d. Based on the value of $r = 0.91$, describe the strength and direction of the linear association. Explain whether this correlation proves that studying more causes higher exam scores.

PART IV — Extended Constructed Response (Question 35)

This question is worth 6 credits. Show all work.

35. A company sells handmade candles. The company's daily revenue R (in dollars) depends on the price p (in dollars) charged per candle according to the function $R(p) = -20p^2 + 400p$.

a. Determine the price per candle that maximizes daily revenue. Use the axis of symmetry formula and show your work.

b. Find the maximum daily revenue. Show your calculation.

c. Determine the prices at which the daily revenue equals \$0, and explain what each of these prices means in context.

d. Create a table of values for $R(p)$ at $p = 0, 5, 10, 15$, and 20. Show all calculations.

e. The company currently charges \$5 per candle. By how much would the daily revenue increase if the company raised the price to the revenue-maximizing price found in part (a)? Show all work.

ANSWER KEY WITH EXPLANATIONS

1. C — $P(h) = 200(2)^h$, because exponential doubling multiplies the starting amount by a constant ratio of 2 each hour. The initial value 200 is the coefficient, and the base 2 represents doubling. Adding (Choice A) would model linear growth, not repeated multiplication.

2. A — $x = 6$. Distribute to get $5x - 10 = 2x + 8$, then subtract $2x$ and add 10 to obtain $3x = 18$, so $x = 6$. Substituting back gives $5(4) = 20$ and $2(6) + 8 = 20$, confirming the solution.

3. D — slope = $-4/3$. Using $(y_2 - y_1)/(x_2 - x_1) = (-6 - 2)/(3 - (-3)) = -8/6 = -4/3$. The negative slope reflects that y decreases as x increases. Reducing the fraction is required for the correct match.

4. B — $(x + 7)(x - 4)$. The factors of -28 that add to $+3$ are $+7$ and -4, giving $(x + 7)(x - 4)$. Expanding confirms $x^2 + 3x - 28$. Sign placement matters: the larger-magnitude factor takes the positive sign to produce $+3x$.

5. D — $f(3) = -2$. Substitute: $-(3)^2 + 4(3) - 5 = -9 + 12 - 5 = -2$. The negative sign applies to the squared term, so $(3)^2 = 9$ becomes -9. Choice B results from incorrectly squaring before applying the negative.

6. C — $x = 2$ and $x = 4$. Rewrite as $x^2 - 6x + 8 = 0$, then factor to $(x - 2)(x - 4) = 0$, giving $x = 2$ and $x = 4$. Both factors are set equal to zero by the zero-product property. The equation must first be set equal to zero before factoring.

7. A — $a_n = 5 - 4(n - 1)$. The first term is 5 and the common difference is -4 (each term decreases by 4). The explicit form $a_1 + d(n - 1)$ gives $5 - 4(n - 1)$. A negative common difference is required because the sequence decreases.

8. B — $x < -4$. Subtract 3 to get $-4x > 16$, then divide by -4, which reverses the inequality sign to $x < -4$. Dividing or multiplying by a negative value always flips the inequality direction. Failing to flip the sign produces Choice A.

9. A — (2, 3). Set $2x - 1 = -x + 5$, giving $3x = 6$, so $x = 2$; substituting yields $y = 2(2) - 1 = 3$. The solution to a system is the point that satisfies both equations. Verification in both equations confirms (2, 3).

10. D — 8% decrease per year. In $V(t) = 8000(0.92)^t$, the base 0.92 equals $1 - 0.08$, indicating an 8% decrease each year. A base less than 1 always signals exponential decay. The percentage rate is the difference between the base and 1.

11. B — $2x^2 + 7x - 8$. Distribute the subtraction: $3x^2 + 5x - 2 - x^2 + 2x - 6$, then combine like terms to get $2x^2 + 7x - 8$. Every term in the second polynomial changes sign. Forgetting to distribute the negative to all terms produces the other choices.

12. C — shifted vertically up 5 units. Adding a constant outside the squared term, $x^2 + 5$, shifts the parabola upward by 5. Vertical shifts come from constants added to the whole function. Horizontal shifts would appear inside the parentheses as $(x - h)$.

13. A — $2x - 3y = 12$. Start with $y = (2/3)x - 4$, multiply through by 3 to clear the fraction ($3y = 2x - 12$), then rearrange to $2x - 3y = 12$. Standard form $Ax + By = C$ requires integer coefficients with x positive. The intercept −4 produces the constant 12 after multiplying.

14. D — $f = -4$. The vertex x-value is $-b/(2a) = -6/2 = -3$, and $f(-3) = 9 - 18 + 5 = -4$. Because $a > 0$, the parabola opens upward and the vertex gives the minimum. The minimum value is the y-coordinate of the vertex.

15. B — irrational because 20 is not a perfect square. The square root of a non-perfect-square integer is irrational, producing a non-terminating, non-repeating decimal. Although $\sqrt{20}$ simplifies to $2\sqrt{5}$, that form still contains an irrational radical. Only perfect squares yield rational square roots.

16. C — Median; the value 92 is an outlier that distorts the mean. The remaining values cluster in the teens, so 92 pulls the mean far above the typical value. The median resists outliers and better represents the center here. The mean is most reliable only when no extreme values are present.

17. D — $m \leq 9$. The total cost is $25 + 15m$, set at or below 160: $25 + 15m \leq 160$, so $15m \leq 135$ and $m \leq 9$. The flat fee is added once and the monthly rate is multiplied by m. Solving the inequality keeps the cost within budget.

18. A — Exponential, with a common ratio of 2. Each y-value is multiplied by 2 (3, 6, 12, 24, 48), which defines exponential growth. A constant ratio between successive outputs indicates an exponential function, not a constant difference. The ratio $6/3 = 2$ confirms the base.

19. C — $r = 0.93$. A strong upward (positive) trend with points tightly clustered near a line yields a correlation coefficient close to +1. A value of 0.93 reflects both strong strength and positive direction. A negative value would indicate a downward trend, which is not the case here.

20. B — $y = 2x + 1$ and $y = 2x - 5$. These lines have the same slope (2) but different y-intercepts, so they are parallel and never intersect. Parallel lines produce a system with no solution. Equal slopes with unequal intercepts is the defining condition.

21. D — (−4, 1). Substituting $x = -4$ gives $y = (1/2)(-4) + 3 = -2 + 3 = 1$, so the point satisfies the equation. A point lies on a line only if its coordinates make the equation true. The other points fail when tested.

22. A — $2(x - 3)(x + 3)$. First factor out the GCF of 2 to get $2(x^2 - 9)$, then recognize $x^2 - 9$ as a difference of two squares, $(x - 3)(x + 3)$. Complete factoring requires removing the GCF before applying the difference-of-squares pattern. The result is fully factored.

23. C — IQR = 12; the spread of the middle 50% of the data values. The interquartile range is $Q3 - Q1 = 22 - 10 = 12$. The IQR measures the range covered by the central half of the data, between the first and third quartiles. It is unaffected by the minimum and maximum.

24. B — $x = 3$ and $x = -5$. Setting each factor to zero, $2x - 6 = 0$ gives $x = 3$, and $x + 5 = 0$ gives $x = -5$. The zeros are the x-values that make the function equal zero. The factor $2x - 6$ must be solved fully rather than read as $x = 6$.

25. From $x - y = 7$, $x = y + 7$. Substitute into $3x + 2y = 4$: $3(y + 7) + 2y = 4 \rightarrow 3y + 21 + 2y = 4 \rightarrow 5y = -17 \rightarrow y = -17/5$. Then $x = -17/5 + 7 = 18/5$. Solution: $(18/5, -17/5)$. Verify: $3(18/5) + 2(-17/5) = 54/5 - 34/5 = 20/5 = 4$ ✓ and $18/5 - (-17/5) = 35/5 = 7$ ✓.

26. The given slope is 1/3, so the perpendicular slope is the negative reciprocal, −3. Using point-slope with $(3, -5)$: $y - (-5) = -3(x - 3) \rightarrow y + 5 = -3x + 9 \rightarrow y = -3x + 4$. Answer: $y = -3x + 4$.

27. Factored form: $f(x) = 2(x + 5)(x - 1)$. Expand: $(x + 5)(x - 1) = x^2 + 4x - 5$, then multiply by 2: $f(x) = 2x^2 + 8x - 10$. Standard form: $f(x) = 2x^2 + 8x - 10$.

28. Quadratic. The first differences (+15, +5, −5, −15) are not constant, but the second differences (−10, −10, −10) are constant, which indicates a quadratic function. The values are symmetric about $t = 2$, so the maximum height is 26 feet, occurring at $t = 2$ seconds.

29. Factor out the GCF $4x$: $4x^3 - 36x = 4x(x^2 - 9)$. Then factor the difference of squares: $x^2 - 9 = (x - 3)(x + 3)$. Completely factored: $4x(x - 3)(x + 3)$.

30. $a_7 - a_3 = 4d$, so $32 - 12 = 4d \rightarrow 20 = 4d \rightarrow d = 5$. Since $a_3 = a_1 + 2d$, $12 = a_1 + 10 \rightarrow a_1 = 2$. Explicit formula: $a_n = 2 + 5(n - 1)$.

31. Graph $y < (1/2)x + 4$ as a dashed line shaded below, and $y \geq -x - 1$ as a solid line shaded above; the solution is the overlapping region. One valid point is $(0, 0)$: $0 < (1/2)(0) + 4 = 4$ ✓ (true), and $0 \geq -(0) - 1 = -1$ ✓ (true). So $(0, 0)$ lies in the solution set.

32. Predicted value: $\hat{y} = 2.5(8) + 30 = 20 + 30 = 50$. Residual = actual − predicted = $60 - 50 = 10$. The residual is positive, which means the actual sales (60) were above the value predicted by the line, so the line underestimated this data point.

33. a. The axis of symmetry is $x = -b/(2a) = 10/2 = 5$. Then $f(5) = 25 - 50 + 21 = -4$, so the vertex is $(5, -4)$ and the minimum value is −4. The assertion is TRUE.

b. Factor: $x^2 - 10x + 21 = (x - 3)(x - 7) = 0$, so the zeros are $x = 3$ and $x = 7$.

c. The parabola opens upward ($a = 1 > 0$). The vertex is $(5, -4)$ and the axis of symmetry is $x = 5$. The x-intercepts are (3, 0) and (7, 0). The y-intercept is (0, 21), found by evaluating $f(0)$.

34. a. The slope of 4 means that for each additional hour studied, a student's exam score is predicted to increase by 4 points.

b. $y = 4(9) + 52 = 36 + 52 = 88$. The predicted score is 88.

c. Predicted: $y = 4(7) + 52 = 80$. Residual = $85 - 80 = 5$. The actual score was above the predicted value.

d. $r = 0.91$ indicates a strong, positive linear association. However, correlation does not prove causation; the relationship does not establish that studying more causes higher scores, since other factors (such as prior knowledge or test conditions) could influence both variables.

35. a. $p = -b/(2a) = -400/(2 \cdot -20) = -400/-40 = 10$. The revenue-maximizing price is \$10 per candle.

b. $R(10) = -20(10)^2 + 400(10) = -2000 + 4000 = 2000$. The maximum daily revenue is \$2,000.

c. Set $R(p) = 0$: $-20p^2 + 400p = 0 \rightarrow -20p(p - 20) = 0 \rightarrow p = 0$ or $p = 20$. A price of \$0 means candles are given away and produce no revenue; a price of \$20 is so high that no candles are sold, also producing no revenue.

d. $p = 0$: $-20(0) + 400(0) = 0$. $p = 5$: $-20(25) + 400(5) = -500 + 2000 = 1500$. $p = 10$: 2000 (from part b). $p = 15$: $-20(225) + 400(15) = -4500 + 6000 = 1500$. $p = 20$: $-20(400) + 400(20) = -8000 + 8000 = 0$.

p: 0, 5, 10, 15, 20

R(p): 0, 1500, 2000, 1500, 0

e. At $5 the revenue is R(5) = 1500; at the optimal price $10 the revenue is R(10) = 2000. Increase = 2000 − 1500 = 500. Daily revenue would increase by $500.