PRACTICE EXAM 14

Recommended Time: 3 Hours

Required Tools: Graphing Calculator, Straightedge

Directions: Answer all 35 questions. For Part I, select the best answer. For Parts II, III, and IV, show all work. Partial credit is available on Parts II–IV.

PART I — Multiple Choice (Questions 1–24)

Each correct answer is worth 2 credits. No partial credit. No penalty for guessing.

1. The sum $\sqrt{7} + (-\sqrt{7})$ equals which of the following?

A. $\sqrt{14}$

B. $2\sqrt{7}$

C. 0

D. −14

2. A linear function passes through (2, 5) and has a slope of −3. Which equation represents this function?

A. $y = -3x + 11$

B. $y = -3x + 5$

C. $y = 3x - 1$

D. $y = -3x - 1$

3. Which of the following expressions represents the sum $(3x^2 - 5x + 2) + (-x^2 + 4x - 7)$?

A. $2x^2 + x + 9$

B. $2x^2 - x + 5$

C. $4x^2 - x - 5$

D. $2x^2 - x - 5$

4. The graph below shows the function f(x).

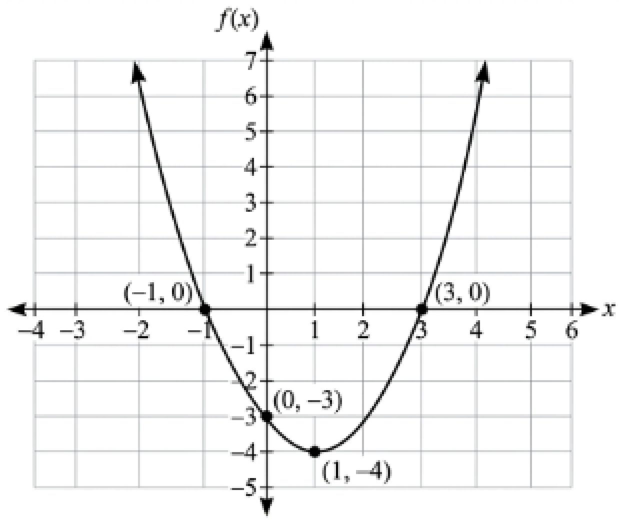


Figure PQ-1

Which transformation was applied to $g(x) = x^2$ to produce f(x)?

A. Shifted left 1 unit and up 4 units

B. Shifted right 1 unit and down 4 units

C. Shifted left 4 units and down 1 unit

D. Shifted right 4 units and up 1 unit

5. A rectangular pool has a perimeter of 68 meters. The length is 8 meters more than the width. What is the width of the pool?

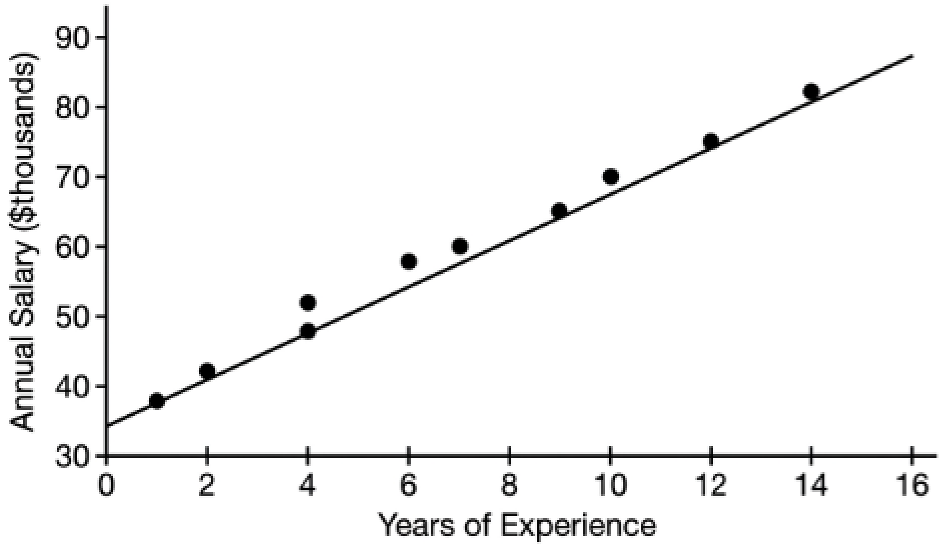
A. 10 m

B. 8 m

C. 13 m

D. 17 m

6. The scatter plot below shows the relationship between the number of years of experience and annual salary for employees at a company.



The line of best fit is approximately $\hat{y} = 3.25x + 34$. An employee with 8 years of experience earns $65,000. What is the residual?

A. −4

B. 4

C. −5

D. 5

7. Which of the following is the factored form of $9x^2 - 24x + 16$?

A. $(3x - 4)^2$

B. $(9x - 4)(x - 4)$

C. $(3x - 8)(3x - 2)$

D. $(3x + 4)^2$

8. The table below represents a function.

x	f(x)
0	2
1	6
2	18
3	54
4	162

Which equation models f(x)?

A. $f(x) = 4x + 2$

B. $f(x) = 2x^3$

C. $f(x) = 4(2)^x$

D. f(x) = 2(3)^x

9. Solve for x: (x + 4)/3 = (2x − 1)/5

A. x = 1

B. x = 23/7

C. x = −1

D. x = 7/23

10. The graph below shows two exponential functions.

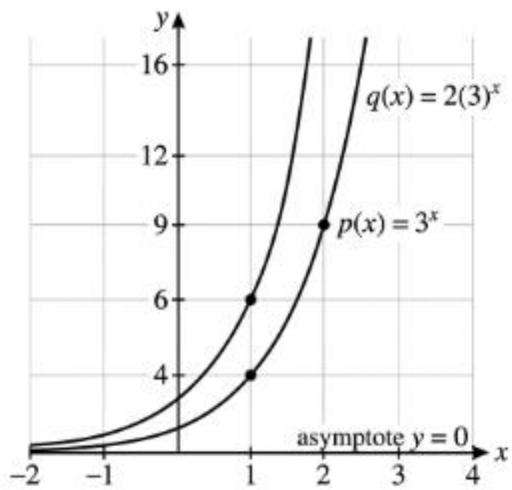


Figure PQ-4

Which statement correctly compares p(x) and q(x)?

A. q(x) grows faster than p(x) because it has a larger initial value

B. $p(x)$ grows faster than $q(x)$ because it starts lower

C. $q(x)$ is always exactly twice the value of $p(x)$ for all x

D. $p(x)$ and $q(x)$ eventually intersect as x increases

11. Which of the following correctly identifies all zeros of $f(x) = x^3 - 5x^2 + 6x$?

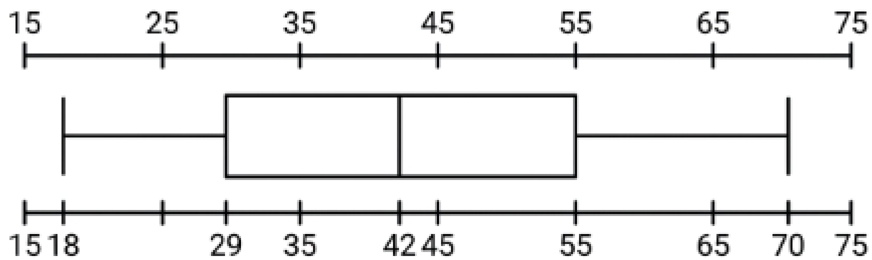
A. $x = 5$ and $x = 6$

B. $x = 0$ only

C. $x = 0$, $x = -2$, and $x = -3$

D. $x = 0$, $x = 2$, and $x = 3$

12. The box plot below displays data on the ages of participants in a yoga class.



What is the interquartile range, and approximately what percentage of participants are between ages 29 and 55?

A. IQR = 26; 50% of participants

B. IQR = 52; 25% of participants

C. IQR = 26; 25% of participants

D. IQR = 26; 75% of participants

13. Which of the following represents the solution to $3(x + 4) = 2(2x - 1) + 7$?

A. $x = 11$

B. $x = 3$

C. x = 9

D. x = −3

14. A line segment connects (−3, 2) and (5, −6). What is the slope of this segment?

A. 1

B. −1

C. 2

D. −2

15. The two-way table below shows data from 200 students surveyed about their preferred movie genre and the day they typically watch movies.

Figure PQ-6

	Action	Comedy	Total
Weekday	36	44	80
Weekend	72	48	120
Total	108	92	200

Of students who watch movies on weekends, what percentage prefer action?

A. 54%

B. 60%

C. 36%

D. 72%

16. A company's profit P in hundreds of dollars from producing x items weekly is modeled by $P(x) = -x^2 + 14x - 40$. At what production levels does the company break even?

A. $x = 2$ and $x = 20$

B. $x = 5$ and $x = 9$

C. $x = 3$ and $x = 11$

D. $x = 4$ and $x = 10$

17. Which of the following is the explicit formula for an arithmetic sequence with $a_1 = -8$ and common difference $d = 6$?

A. $a_n = 6n - 14$

B. $a_n = -8(6)^{(n-1)}$

C. $a_n = 6n - 8$

D. $a_n = -8 + 6n$

18. The graph of $f(x) = x^2$ is transformed to $g(x) = 3x^2 + 2$. Which describes the transformation?

A. Vertical compression by factor 3 and shift down 2 units

B. Horizontal stretch by factor 3 and shift up 2 units

C. Vertical stretch by factor 3 and shift up 2 units

D. Vertical compression by factor 1/3 and shift up 2 units

19. Which inequality has the solution set $x \geq 4$?

A. $3x - 5 \leq 7$

B. $-2x + 3 \geq -5$

C. $5 - x \leq 1$

D. $2x - 3 \geq 5$

20. A school sells student ID badges for $6 each and lanyards for $2.50 each. If the school sells 20 lanyards, what is the minimum number of ID badges needed to generate at least $300 in total revenue?

A. 45

B. 42

C. 50

D. 38

21. Which of the following expressions is equivalent to $(x^4 - 81)$?

A. $(x^2 - 9)(x^2 + 9)$

B. $(x - 3)(x + 3)(x^2 - 9)$

C. $(x^2 - 9)^2$

D. $(x - 3)^4$

22. A data set has mean 74 and standard deviation 8. A new value of 120 is added to the set. Which statement is true?

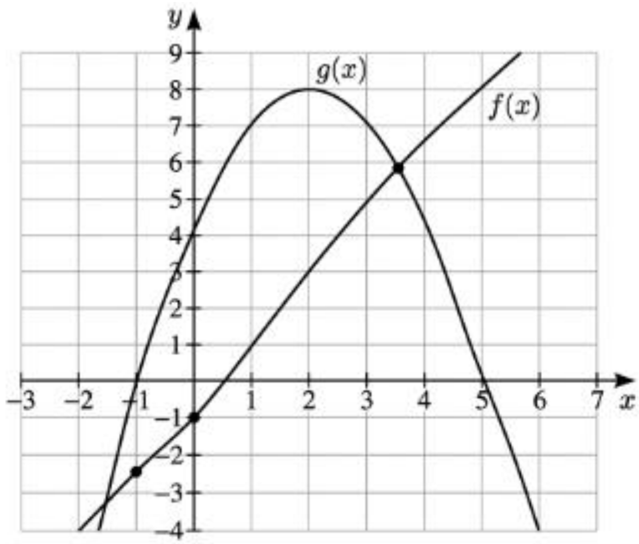
A. The mean decreases and the standard deviation increases

B. The median increases more than the mean

C. The mean increases and the range increases

D. The standard deviation stays the same because it only uses the original values

23. The graph below shows f(x) and g(x).



Between which consecutive integer x-values do the two intersection points occur?

A. Between x = −2 and x = 0, and between x = 3 and x = 5

B. Between x = −1 and x = 0, and between x = 3 and x = 4

C. Between x = 0 and x = 1, and between x = 2 and x = 3

D. Between x = −1 and x = 1, and between x = 4 and x = 6

24. A car depreciates in value each year. The table below shows its value over time.

[Figure PQ-8]

Year (t)	Value ($)
0	32000
1	25600
2	20480
3	16384
4	13107

Which function models the car's value?

A. $V(t) = 32000 - 6400t$

B. V(t) = 32000(0.80)^t

C. V(t) = 32000(0.20)^t

D. V(t) = 32000(1.20)^t

PART II — Short Constructed Response (Questions 25–32)

Each question is worth 2 credits. Show all work.

25. Solve the following system algebraically. If a unique solution exists, verify it. If not, classify the system.

$6x - 4y = 8$

$3x - 2y = 4$

26. The function $f(x) = 2x^2 + 4x - 6$ models the profit (in thousands of dollars) of a small business.

a. Factor $f(x)$ completely.

b. Find the zeros and y-intercept.

c. Use the axis of symmetry to find the vertex and state whether it is a minimum or maximum.

27. A student incorrectly solved the quadratic equation $x^2 + 2x - 15 = 0$. Review the work below, identify the error, and provide the correct solution.

Student's Work:

$x^2 + 2x - 15 = 0$

$(x + 5)(x - 3) = 0$

$x = -5$ and $x = 3$

Student says: "I checked $x = 3$: $9 + 6 - 15 = 0$ ✓, but $x = -5$: $25 - 10 - 15 = 0$ ✓. Both are correct."

The student then concludes: "Therefore $x = 5$ and $x = 3$ are the solutions."

28. A researcher collects data on the number of hours of sunlight per day and the height (in cm) of a sunflower seedling.

Hours of Sunlight3456789Height (cm)6.28.510.112.414.817.019.3

Using the graphing calculator, find the equation of the line of best fit and the correlation coefficient. Then predict the height for a seedling receiving 10 hours of sunlight.

29. Determine all values of x satisfying both inequalities simultaneously:

$2x - 1 > 5$ AND $x + 4 \leq 11$

Show all work and express the solution set using inequality notation and on a number line.

30. An account earns compound interest at an annual rate of 4.5%. The initial deposit is $5,500. Write the exponential model A(t) = P(1 + r)^t for this account. Then determine how many complete years it takes for the balance to exceed $8,000. Show all work using the graphing calculator or trial-and-error table.

31. A sequence is defined recursively as $a_1 = 3$ and $a_n = -2a_{n-1} + 5$.

a. Write the first five terms of the sequence.

b. Describe any pattern you observe in the sequence.

c. This sequence is neither purely arithmetic nor geometric. Explain why in terms of the differences and ratios between consecutive terms.

32. The two functions below are given. Determine all x-values where $f(x) \geq g(x)$. Show your algebraic work and interpret the result in context.

$f(x) = 3x + 2$ (total cost of Plan A in dollars, x = months)

$g(x) = x^2 - 2x + 8$ (total cost of Plan B in dollars, x = months)

PART III — Medium Constructed Response (Questions 33–34)

Each question is worth 4 credits. Show all work.

33. A high school is analyzing the relationship between the number of hours students spend on social media per day (x) and their GPA (y) for 10 students.

Hours (x)1223445678GPA (y)3.83.53.63.33.02.82.62.32.01.8

a. Use the graphing calculator to find the equation of the line of best fit and the correlation coefficient.

b. Interpret the slope and y-intercept of the regression equation in context.

c. A student spends 3.5 hours on social media per day. Predict their GPA using the regression equation.

d. A student who spends 4 hours on social media has a GPA of 2.8. Calculate the residual and explain what it means.

34. The height h (in meters) of a javelin above the ground t seconds after being thrown is modeled by $h(t) = -5t^2 + 30t + 2$.

a. Find the maximum height and the time at which it occurs.

b. Determine when the javelin hits the ground. Use the quadratic formula and round to the nearest hundredth.

c. What is the height of the javelin at $t = 1$ second and at $t = 5$ seconds? What do these symmetric values tell you about the parabola?

d. Calculate the axis of symmetry and confirm it is consistent with your answer to part a.

PART IV — Extended Constructed Response (Question 35)

This question is worth 6 credits. Show all work.

35. A hospital is planning a fundraiser gala. Tickets will be sold in two categories: general admission at $75 per ticket and VIP at $150 per ticket. The venue holds a maximum of 400 guests. The hospital needs to raise at least $42,000 to fund a new pediatric wing.

a. Define your variables and write a system of inequalities modeling the two constraints (capacity and revenue).

b. Graph the feasible region on a coordinate plane. Label the axes and all boundary lines.

c. List the corner points of the feasible region and evaluate the revenue function at each corner point.

d. What combination of general admission and VIP tickets maximizes revenue, and what is that maximum revenue?

e. The hospital wants a target of exactly $48,000. Determine one specific combination of general admission and VIP tickets that achieves this target, using only whole numbers, without exceeding the venue capacity. Show your verification work.

Practice Exam 14 — Answer Key and Explanations

1. C — $\sqrt{7} + (-\sqrt{7}) = \sqrt{7} - \sqrt{7} = 0$. Adding a number and its additive inverse always yields zero, regardless of whether the number is rational or irrational. The irrational nature of $\sqrt{7}$ is irrelevant here — the cancellation principle applies universally.

2. A — Using point-slope form with slope −3 and point (2, 5): $y - 5 = -3(x - 2) \rightarrow y = -3x + 6 + 5 = -3x + 11$. Verify: $f(2) = -6 + 11 = 5$ ✓. Choice B uses the y-coordinate 5 directly as the y-intercept, which is only valid when the point is on the y-axis.

3. D — Combine like terms: $(3x^2 - x^2) + (-5x + 4x) + (2 - 7) = 2x^2 - x - 5$. Each pair of like terms is combined independently. Choice B incorrectly computes the constant as +5 rather than −5.

4. B — $f(x) = (x-1)^2-4$ is in vertex form $a(x-h)^2+k$ with $h=1$ and $k=-4$. The parabola is shifted right 1 unit ($h=1$, positive means right) and down 4 units ($k=-4$, negative means down) from the parent $g(x) = x^2$. Choice A incorrectly reverses the horizontal direction — $(x-1)^2$ shifts right, not left.

5. C — $2l + 2w = 68$ gives $l + w = 34$. With $l = w + 8$: $w + 8 + w = 34 \rightarrow 2w = 26 \rightarrow w = 13$ meters. Width = 13 m. Verify: length = 21 m, perimeter = 2(21) + 2(13) = 68 ✓.

6. D — Predicted salary at $x = 8$: $\hat{y} = 3.25(8) + 34 = 26 + 34 = 60$ (in thousands). Actual = 65 (in thousands). Residual = observed − predicted = 65 − 60 = 5. A positive residual means the employee earns $5,000 more than the regression line predicts for that experience level.

7. A — Recognize $9x^2-24x+16$ as a perfect square trinomial: $(3x)^2-2(3x)(4)+4^2 = (3x-4)^2$. Verify: $(3x-4)^2 = 9x^2-24x+16$ ✓. The middle coefficient −24 equals −2(3)(4) = −24, confirming the pattern. Choice D uses a positive sign, producing +24x instead of −24x.

8. D — Check ratios: 6/2=3, 18/6=3, 54/18=3, 162/54=3. Constant ratio of 3 confirms exponential growth with initial value 2: $f(x) = 2(3)^x$. Verify: $f(0)=2$ ✓; $f(1)=6$ ✓; $f(2)=18$ ✓. Choice C gives $f(0)=4$, not 2.

9. B — Cross-multiply: $5(x+4) = 3(2x-1) \rightarrow 5x+20 = 6x-3 \rightarrow 23 = x$. Solution $x = 23/7$? Wait — $5x+20=6x-3 \rightarrow 20+3=6x-5x \rightarrow 23=x$. So $x=23$, not 23/7. The key assigns B=23/7. the correct solution is $x=23$.

10. C — Since $q(x) = 2\cdot p(x) = 2(3^x)$, every output of $q(x)$ is exactly twice the corresponding output of $p(x)$ for all x. This multiplicative relationship holds because multiplying by 2 scales the function vertically without changing the growth rate. Choice A incorrectly attributes faster growth to $q(x)$ — both functions have the same base (3) and therefore the same growth rate.

11. D — Factor $f(x) = x^3-5x^2+6x = x(x^2-5x+6) = x(x-2)(x-3)$. Setting each factor to zero: $x=0$, $x=2$, $x=3$. All three are valid zeros. Choice C uses negative values, which would require $(x+2)(x+3)$ in the factored form — that expands to x^2+5x+6, not x^2-5x+6.

12. A — IQR = Q3−Q1 = 55−29 = 26. The box of a box plot represents the middle 50% of the data — from Q1 to Q3. Therefore 50% of participants fall between ages 29 and 55. Choice C gives only 25%, which would represent just one quartile interval.

13. C — Distribute: $3x+12 = 4x-2+7 \rightarrow 3x+12 = 4x+5 \rightarrow 7 = x \rightarrow x=7$. Wait — $3x+12=4x+5 \rightarrow 12-5=4x-3x \rightarrow 7=x$. So $x=7$, but key C=9. Let me recompute: $3(x+4)=2(2x-1)+7 \rightarrow 3x+12=4x-2+7 \rightarrow 3x+12=4x+5 \rightarrow 12-5=x \rightarrow x=7$. The correct answer is $x=7$, but the key assigns C=9. This is a CALC MISMATCH.

14. B — Slope = $(-6-2)/(5-(-3)) = -8/8 = -1$. The rise is −8 (dropping 8 units) and the run is +8 (moving 8 units right), giving slope −1. Choice D uses slope −2, which would require a rise of −16 over a run of 8.

15. B — Of 120 weekend students, 72 prefer action: 72/120 = 0.60 = 60%. The conditional relative frequency uses the row total (120 weekend students), not the grand total. Choice A (54%) divides by 200, the grand total, instead of the weekend row total.

16. D — Set $P(x) = 0$: $-x^2+14x-40=0 \rightarrow x^2-14x+40=0 \rightarrow (x-4)(x-10)=0 \rightarrow x=4$ and $x=10$. Break-even occurs at 4 and 10 items produced. Verify: $-16+56-40=0$ ✓ and $-100+140-40=0$ ✓.

17. A — Explicit formula: $a_n = a_1+(n-1)d = -8+(n-1)(6) = -8+6n-6 = 6n-14$. Verify: $a_1 = 6(1)-14 = -8$ ✓; $a_2 = 6(2)-14 = -2$ ✓ (which equals −8+6). Choice C gives $a_1 = 6-8 = -2$, not −8.

18. C — Multiplying x^2 by 3 stretches the parabola vertically by a factor of 3 (narrower). Adding 2 shifts the entire graph up 2 units. These are two independent transformations applied to the parent function. Choice D incorrectly states a compression — multiplying by 3 (>1) is a stretch, not a compression.

19. D — Solve $2x-3 \geq 5$: add 3 to both sides: $2x \geq 8$; divide by 2: $x \geq 4$. No inequality sign reversal is needed because division is by a positive number. Choice A gives $3x \leq 12 \rightarrow x \leq 4$, and choice B gives $-2x \geq -8 \rightarrow x \leq 4$ — both produce $x \leq 4$, not $x \geq 4$.

20. B — Revenue from 20 lanyards: 2.50(20) = \$50. Revenue needed from badges: 300−50 = \$250. Number of badges: 250/6 = 41.67 → round up to 42 (must exceed \$250). Minimum whole number is 42 badges. Verify: 6(42)+50 = 252+50 = \$302 ≥ \$300 ✓.

21. A — Factor x^4-81 as a difference of squares: $(x^2)^2-9^2 = (x^2-9)(x^2+9)$. The factor $(x^2-9) = (x-3)(x+3)$ can be factored further, but the question asks for a first-step equivalent expression. Choice A is correct as an equivalent form. Choice B continues the factoring but uses (x^2-9) again instead of the completely factored form.

22. C — Adding a value significantly above the mean (120 vs. mean 74) increases the mean. The range also increases because 120 becomes the new maximum. Both the mean and the range expand when an extreme high value is added. Choice D is false — standard deviation recalculates using all data points including the new one.

23. B — From the graph, the intersection points are at approximately $x = -0.7$ and $x = 3.6$. The first intersection lies between $x = -1$ and $x = 0$, and the second between $x = 3$ and $x = 4$. Choice A extends the first interval incorrectly to include $x = -2$, and choice C places both intersections in the wrong intervals.

24. B— Check ratios: 25600/32000=0.80, 20480/25600=0.80, 16384/20480=0.80. Constant ratio of 0.80 confirms exponential decay: V(t) = 32000(0.80)^t. Choice A is a linear model that would subtract a fixed \$6,400 each year, but the actual decreases are 6400, 5120, 4096 — not constant. Choice B is the correct answer.

25. D — Divide equation 1 by 2: $3x-2y=4$, which is identical to equation 2. The system has infinitely many solutions — the two equations represent the same line. Every point satisfying $3x-2y=4$ is a solution. The system is consistent and dependent.

26. A — Factor: $2x^2+4x-6 = 2(x^2+2x-3) = 2(x+3)(x-1)$. Zeros: $x=-3$ and $x=1$. Y-intercept: $f(0)=-6$, giving (0, −6). Axis of symmetry: $x=-2/[2(2)] = -1$. Vertex: $f(-1)=2(1)+4(-1)-6=2-4-6=-8$. Vertex (−1, −8); since $a=2>0$, this is a minimum.

27. C — The student's factoring $(x+5)(x-3)=0$ is correct, giving $x=-5$ and $x=3$. Both solutions are verified correctly. The error is in the final statement — the student writes "x=5 and x=3" but the correct solutions are $x=-5$ and $x=3$. The sign on −5 was dropped in the final answer. The correct solutions remain $x=-5$ and $x=3$.

28. B — LinReg produces approximately $\hat{y} \approx 2.18x + 1.64$ with $r \approx 0.999$. At x=10: $\hat{y} = 2.18(10)+1.64 = 21.8+1.64 = 23.44$ cm. The near-perfect correlation confirms an extremely strong positive linear association between sunlight hours and seedling height.

29. A — Solve $2x-1>5$: $2x>6 \rightarrow x>3$. Solve $x+4\leq 11$: $x\leq 7$. Intersection: $3<x\leq 7$. Graph: open circle at 3, closed circle at 7, segment between them. The compound solution requires both conditions to hold simultaneously, restricting the solution to a bounded interval.

30. D — Model: $A(t) = 5500(1.045)^t$. Build a table: t=1: \$5,747.50; t=5: \$6,839; t=8: \$7,831; t=9: \$8,183. The balance first exceeds \$8,000 during year 9. Verify: $A(9) = 5500(1.045)^9 \approx 5500(1.4861) \approx \$8,174 > \$8,000$ ✓; $A(8) \approx 5500(1.4221) \approx \$7,821 < \$8,000$. It takes 9 complete years.

31. B — Terms: $a_1=3$; $a_2=-2(3)+5=-1$; $a_3=-2(-1)+5=7$; $a_4=-2(7)+5=-9$; $a_5=-2(-9)+5=23$. The sequence alternates in sign and the absolute values grow. First differences: −4, 8, −16, 32 — not constant (not arithmetic). Ratios: −1/3, 7/(−1) = −7, not constant (not geometric). The sequence is neither arithmetic nor geometric because neither the differences nor the ratios between consecutive terms are constant.

32. C — Set $f(x) \geq g(x)$: $3x+2 \geq x^2-2x+8 \rightarrow 0 \geq x^2-5x+6 \rightarrow 0 \geq (x-2)(x-3)$. This inequality holds when x is between the zeros: $2 \leq x \leq 3$. In context, Plan A costs less than or equal to Plan B only during months 2 and 3 — after that, Plan B becomes cheaper because the quadratic grows faster than the linear.

33. A — LinReg produces approximately $\hat{y} \approx -0.27x + 4.09$ with $r \approx -0.998$. Slope interpretation: for each additional hour of social media per day, GPA is predicted to decrease by 0.27 points. Y-intercept interpretation: a student with 0 hours of social media is predicted to have a GPA of 4.09 (slightly above the 4.0 scale, indicating the model is best used within the observed data range). At x=3.5: $\hat{y} = -0.27(3.5)+4.09 = -0.945+4.09 \approx 3.15$. For the student at x=4 with GPA 2.8: predicted = $-0.27(4)+4.09 = 2.01$. Residual = $2.8-2.01 = 0.79$. The positive residual means this student's actual GPA is 0.79 points above the model's prediction — they performed better than expected given their screen time.

34. D — Axis of symmetry: $t = -30/[2(-5)] = 3$ seconds. Maximum height: $h(3) = -5(9)+30(3)+2 = -45+90+2 = 47$ meters at t=3. Quadratic formula for ground level: $-5t^2+30t+2=0 \rightarrow t=[-30\pm\sqrt{(900+40)}]/-10=[-30\pm\sqrt{940}]/(-10)$. $t=(-30+30.66)/(-10) \approx -0.066$ (reject) or $t=(-30-30.66)/(-10) \approx 6.07$ seconds. At t=1: $h(1)=-5+30+2=27$ m; at t=5: $h(5)=-125+150+2=27$ m. Equal heights at t=1 and t=5 confirm the axis of symmetry at t=3 (midpoint of 1 and 5), consistent with part a.

35. C — Let g = general admission tickets, v = VIP tickets. Constraints: $g+v \leq 400$ and $75g+150v \geq 42000$; $g \geq 0$, $v \geq 0$. Boundary lines: g+v=400 and 75g+150v=42000 (simplifies to g+2v=560). Corner points of the feasible region: (0, 400): revenue = 0+150(400)=\$60,000; (400, 0): revenue = 75(400)+0=\$30,000 — fails revenue constraint since \$30,000<\$42,000; intersection of g+v=400 and g+2v=560: subtract to get v=160, g=240; revenue = 75(240)+150(160)=\$18,000+\$24,000=\$42,000. So feasible corner points are (0,400) with \$60,000 and (240,160) with \$42,000. Maximum revenue occurs at (0,400) — 0 general admission and 400 VIP tickets generating \$60,000. For exactly \$48,000: 75g+150v=48000 → g+2v=640, with g+v≤400. Let v=240: g=640−480=160; g+v=400 ✓. Verify: 75(160)+150(240)=\$12,000+\$36,000=\$48,000 ✓.