PRACTICE SET 10: DISCRETE MATH AND MATHEMATICAL REASONING

1. The next term in the sequence 2, 4, 8, 16, … is:

A. 20

B. 24

C. 32

D. 18

2. The union of sets A = {1, 2, 3} and B = {3, 4, 5} is:

A. {1, 2, 3, 4, 5}

B. {3}

C. {1, 2, 4, 5}

D. {1, 2, 3}

3. A sequence where each term is obtained by adding a constant to the previous term is called:

A. geometric

B. exponential

C. Fibonacci

D. arithmetic

4. The intersection of sets A = {1, 2, 3, 4} and B = {3, 4, 5, 6} is:

A. {1, 2}

B. {3, 4}

C. {5, 6}

D. {1, 2, 3, 4, 5, 6}

5. The statement "If it rains, then the ground is wet" has contrapositive:

A. If the ground is not wet, then it did not rain

B. If the ground is wet, then it rained

C. If it does not rain, then the ground is not wet

D. It rains and the ground is wet

6. The common difference in the arithmetic sequence 5, 9, 13, 17, … is:

A. 5

B. 3

C. 2

D. 4

7. The cardinality of the set {a, b, c, d, e} is:

A. 4

B. 5

C. 6

D. 10

8. A Venn diagram is used to show:

A. time sequences

B. physical dimensions

C. set relationships

D. probability distributions

9. The next term in 1, 3, 6, 10, 15, … is:

A. 18

B. 19

C. 20

D. 21

10. The complement of set A in universe U is:

A. elements in U not in A

B. elements in A not in U

C. elements in both A and U

D. elements in neither

11. An arithmetic sequence has first term 3 and common difference 4. The 5th term is:

A. 15

B. 19

C. 20

D. 23

12. "All dogs are mammals" is an example of:

A. a question

B. a specific case

C. an exception

D. a universal statement

13. The sum of the first 5 terms of the arithmetic sequence 2, 4, 6, 8, 10 is:

A. 30

B. 25

C. 35

D. 40

14. In logic, the negation of "p and q" is:

A. p and not q

B. p or q

C. not p or not q

D. not p and not q

15. A geometric sequence has first term 2 and common ratio 3. The 4th term is:

A. 12

B. 18

C. 27

D. 54

16. If $A = \{1, 2\}$ and $B = \{a, b\}$, the Cartesian product $A \times B$ has how many elements?

A. 2

B. 4

C. 6

D. 8

17. Inductive reasoning moves from:

A. specific observations to general conclusions

B. general rules to specific cases

C. false to true

D. right to left

18. The first 5 terms of the Fibonacci sequence are:

A. 0, 1, 2, 3, 4

B. 1, 2, 3, 4, 5

C. 1, 1, 2, 4, 8

D. 1, 1, 2, 3, 5

19. The next term in 3, 6, 12, 24, … is:

A. 30

B. 36

C. 48

D. 60

20. If p is true and q is false, "p or q" is:

A. true

B. false

C. undefined

D. sometimes true

21. The number of subsets of a set with 3 elements is:

A. 3

B. 6

C. 7

D. 8

22. A set containing no elements is called:

A. empty set

B. universal set

C. infinite set

D. power set

23. The sum of the first 10 positive integers is:

A. 50

B. 45

C. 55

D. 100

24. The next term in 5, 10, 20, 40, … is:

A. 60

B. 80

C. 100

D. 50

25. Deductive reasoning moves from:

A. general rules to specific conclusions

B. specific to general

C. guess to answer

D. false to true

26. If A = {1, 2, 3} and B = {4, 5}, A and B are:

A. equal

B. equivalent

C. subsets

D. disjoint

27. Estimate 198 + 403 to the nearest hundred:

A. 500

B. 600

C. 700

D. 800

28. The common ratio of the geometric sequence 3, 9, 27, 81 is:

A. 1

B. 2

C. 3

D. 4

29. In set notation, $\in$ means:

A. subset of

B. union

C. intersection

D. element of

30. The next term in 100, 90, 80, 70, … is:

A. 60

B. 50

C. 65

D. 55

31. An arithmetic sequence starts at 10 with common difference -3. The 4th term is:

A. 1

B. 4

C. 7

D. -2

32. If $p \rightarrow q$ and p is true, then:

A. q is false

B. p is false

C. q is true

D. both are false

33. The nth term formula for an arithmetic sequence is:

A. $a_n = a_1 \cdot r^n$

B. $a_n = a_1/n$

C. $a_n = a_1 + n$

D. $a_n = a_1 + (n - 1)d$

34. The number of elements in $A \cup B$ when $|A| = 5$, $|B| = 7$, $|A \cap B| = 2$ is:

A. 10

B. 12

C. 14

D. 16

35. The next term in 1, 4, 9, 16, 25, … is:

A. 30

B. 32

C. 35

D. 36

36. A tautology is a statement that is:

A. always false

B. always true

C. sometimes true

D. both true and false

37. The next term in the Fibonacci sequence 1, 1, 2, 3, 5, 8, … is:

A. 13

B. 11

C. 14

D. 16

38. Estimate 29×41:

A. 800

B. 900

C. 1,000

D. 1,200

39. The set $\{x : x > 0 \text{ and } x < 5, x \text{ is an integer}\}$ equals:

A. {0, 1, 2, 3, 4, 5}

B. {1, 2, 3, 4, 5}

C. {1, 2, 3, 4}

D. {0, 1, 2, 3, 4}

40. A statement and its contrapositive have the same:

A. truth value

B. subject

C. length

D. conclusion order

41. The sum of the arithmetic sequence 2 + 4 + 6 + 8 + 10 is:

A. 20

B. 25

C. 30

D. 40

42. If $A \subseteq B$ and $B \subseteq A$, then:

A. $A = B$

B. $A \neq B$

C. A is empty

D. B is empty

43. The nth term of a geometric sequence is:

A. $a_n = a_1 + nd$

B. $a_n = a_1 + d$

C. $a_n = a_1 \cdot n$

D. $a_n = a_1 \cdot r^{(n-1)}$

44. The next term in 2, 5, 10, 17, 26, … is:

A. 34

B. 37

C. 40

D. 45

45. A counterexample disproves:

A. a general claim

B. a specific fact

C. a sequence

D. a set

46. The number of elements in {a, b, c} × {1, 2} is:

A. 2

B. 3

C. 5

D. 6

47. Estimate 997 + 501:

A. 1,000

B. 1,400

C. 1,500

D. 2,000

48. The intersection of {1, 2, 3} and {2, 3, 4} is:

A. {1, 2}

B. {2, 3}

C. {3, 4}

D. {1, 2, 3, 4}

49. If the first term of an arithmetic sequence is 7 and the 10th term is 34, the common difference is:

A. 2

B. 2.5

C. 4

D. 3

50. The next term in 81, 27, 9, 3, … is:

A. 1

B. 0

C. 1/2

D. 1/3

PRACTICE SET 10: ANSWER KEY AND EXPLANATIONS

1. C — 32. Each term doubles the previous one, so 16 × 2 = 32. This is a geometric sequence with common ratio 2.
2. A — {1, 2, 3, 4, 5}. The union combines all unique elements from both sets. Elements appearing in both sets are listed only once.
3. D — arithmetic. An arithmetic sequence adds a constant (the common difference) to each term to produce the next. Geometric sequences multiply by a constant instead.
4. B — {3, 4}. The intersection contains only elements that appear in both sets. Only 3 and 4 belong to both A and B.
5. A — If the ground is not wet, then it did not rain. The contrapositive of "if p then q" is "if not q then not p," and it is logically equivalent to the original statement.
6. D — 4. Each term is 4 more than the previous: 9 − 5 = 4, 13 − 9 = 4, etc. This is the common difference of the arithmetic sequence.
7. B — 5. Cardinality counts the number of elements in a set, and {a, b, c, d, e} has exactly 5 distinct elements.
8. C — set relationships. Venn diagrams use overlapping circles to visualize unions, intersections, and complements of sets.
9. D — 21. The differences between consecutive terms are 2, 3, 4, 5, 6, so the next term is 15 + 6 = 21. This is the triangular number sequence.
10. A — elements in U not in A. The complement of A consists of all elements in the universal set that are not in A.
11. B — 19. Using $a_n = a_1 + (n - 1)d$, $a_5 = 3 + 4(4) = 3 + 16 = 19$.
12. D — a universal statement. "All dogs are mammals" applies to every member of a category, which makes it universal.
13. A — 30. Adding 2 + 4 + 6 + 8 + 10 = 30. Arithmetic series sums can be calculated by direct addition or by the formula $S = n(a_1 + a_n)/2$.
14. C — not p or not q. By De Morgan's law, the negation of "p and q" is "not p or not q." De Morgan's laws convert conjunctions to disjunctions under negation.

15. D — 54. Using $a_n = a_1 \cdot r^{(n-1)}$, $a_4 = 2 \cdot 3^3 = 2 \cdot 27 = 54$.

16. B — 4. The Cartesian product has $|A| \times |B| = 2 \times 2 = 4$ ordered pairs: (1,a), (1,b), (2,a), (2,b).

17. A — specific observations to general conclusions. Inductive reasoning builds general rules from patterns observed in specific cases. Deductive reasoning moves in the opposite direction.

18. D — 1, 1, 2, 3, 5. The Fibonacci sequence starts with two 1's, and each subsequent term is the sum of the two preceding terms.

19. C — 48. Each term doubles the previous: $24 \times 2 = 48$.

20. A — true. In logic, "p or q" is true whenever at least one operand is true. Since p is true, the disjunction is true regardless of q.

21. D — 8. The number of subsets of a set with n elements is 2^n, so $2^3 = 8$.

22. A — empty set. The empty set (denoted $\emptyset$ or $\{\}$) contains no elements and is a subset of every set.

23. C — 55. The sum of the first n positive integers is $n(n+1)/2$, so $10(11)/2 = 55$.

24. B — 80. Each term doubles the previous: $40 \times 2 = 80$.

25. A — general rules to specific conclusions. Deductive reasoning applies general principles to derive specific truths. If the general rule is valid, the specific conclusion must also be valid.

26. D — disjoint. Two sets are disjoint when they share no elements in common, which is the case for $\{1, 2, 3\}$ and $\{4, 5\}$.

27. B — 600. Rounding 198 to 200 and 403 to 400 gives $200 + 400 = 600$. Estimation rounds to convenient values before calculating.

28. C — 3. Each term is 3 times the previous: $9/3 = 3$, $27/9 = 3$, $81/27 = 3$. This is the common ratio of the geometric sequence.

29. D — element of. The symbol $\in$ indicates set membership, read as "is an element of."

30. A — 60. Each term decreases by 10, so $70 - 10 = 60$. This is an arithmetic sequence with common difference -10.

31. A — 1. Using $a_n = a_1 + (n - 1)d$, $a_4 = 10 + 3(-3) = 10 - 9 = 1$.

32. C — q is true. This is modus ponens: if $p \rightarrow q$ and p is true, then q must be true. This is one of the most basic valid logical inferences.

33. D — $a_n = a_1 + (n - 1)d$. This is the standard formula for the nth term of an arithmetic sequence, where a_1 is the first term and d is the common difference.

34. A — 10. Using $|A \cup B| = |A| + |B| - |A \cap B| = 5 + 7 - 2 = 10$. This is the inclusion-exclusion principle, which avoids double-counting shared elements.

35. D — 36. The sequence shows consecutive perfect squares: 1^2, 2^2, 3^2, 4^2, 5^2, so the next is $6^2 = 36$.

36. B — always true. A tautology is a compound statement that is true under every possible assignment of truth values to its components. The statement "p or not p" is a classic tautology.

37. A — 13. Each Fibonacci term is the sum of the two preceding terms: $5 + 8 = 13$.

38. D — 1,200. Rounding 29 to 30 and 41 to 40 gives $30 \times 40 = 1{,}200$. Estimation simplifies calculations while preserving rough accuracy.

39. C — {1, 2, 3, 4}. The integers strictly greater than 0 and strictly less than 5 are 1, 2, 3, and 4. Strict inequalities exclude the boundary values.

40. A — truth value. A statement and its contrapositive are logically equivalent, meaning they are either both true or both false.

41. C — 30. Adding $2 + 4 + 6 + 8 + 10 = 30$. This can also be computed using the arithmetic series sum formula.

42. A — A = B. If each set is a subset of the other, they must contain exactly the same elements, which is the definition of set equality.

43. D — $a_n = a_1 \cdot r^{(n-1)}$. This is the standard formula for the nth term of a geometric sequence, where a_1 is the first term and r is the common ratio.

44. B — 37. The differences are 3, 5, 7, 9, 11, so the next term is $26 + 11 = 37$. This sequence follows the pattern $a_n = n^2 + 1$.

45. A — a general claim. A counterexample is a specific case that proves a universal statement false by showing at least one instance where it fails.

46. D — 6. The Cartesian product has $3 \times 2 = 6$ ordered pairs.

47. C — 1,500. Rounding 997 to 1,000 and 501 to 500 gives $1{,}000 + 500 = 1{,}500$.

48. B — {2, 3}. Only 2 and 3 appear in both sets, making them the intersection.

49. D — 3. Using $a_n = a_1 + (n - 1)d$ with $a_{10} = 34$, $7 + 9d = 34$, so $9d = 27$ and $d = 3$.

50. A — 1. Each term is 1/3 of the previous: $3 \times (1/3) = 1$. This is a geometric sequence with common ratio 1/3.