Quantitative Reasoning

Numerical Calculations

1. A bread recipe calls for $3\frac{1}{4}$ cups of flour. If you only have $2\frac{1}{8}$ cups, how much more flour is needed?

A. $1\frac{1}{8}$

B. $1\frac{1}{4}$

C. $1\frac{3}{8}$

D. $1\frac{3}{4}$

E. $5\frac{3}{4}$

(A) $3\frac{1}{4} - 2\frac{1}{8} = \frac{13}{4} - \frac{17}{8} = \frac{26}{8} - \frac{17}{8} = \frac{9}{8} = 1\frac{1}{8}$ more cups of flour.

2. Jack lives $6\frac{1}{2}$ miles from the library. If he walks $\frac{1}{3}$ of the way and takes a break, what is the remaining distance to the library?

A. $2\frac{1}{6}$ miles

B. 4 miles

C. $4\frac{1}{3}$ miles

D. $5\frac{5}{6}$ miles

E. $6\frac{1}{6}$ miles

(C) $\frac{1}{3}$ of $6\frac{1}{2}$ miles is $\frac{1}{3} \times 6\frac{1}{2} = \frac{1}{3} \times \frac{13}{2} = \frac{13}{6}$ miles walked. The remaining distance is $6\frac{1}{2} - \frac{13}{6} = \frac{13}{2} - \frac{13}{6} = \frac{39}{6} - \frac{13}{6} = \frac{26}{6} = 4\frac{1}{3}$ miles.

3. The sum of 2 feet $2\frac{1}{2}$ inches, 4 feet $3\frac{3}{8}$ inches, and 3 feet $9\frac{3}{4}$ inches is

A. 9 feet $\frac{7}{8}$ inches.

B. 9 feet $9\frac{5}{8}$ inches.

C. 10 feet $\frac{5}{8}$ inches.

D. 10 feet $2\frac{5}{8}$ inches.

E. 10 feet $3\frac{5}{8}$ inches.

(E) First, add the number of feet together and then add the number of inches: 2 ft + 4 ft + 3 ft = 9 ft. Then,

$$2\frac{1}{2}\text{ in} + 3\frac{3}{8}\text{ in} + 9\frac{3}{4}\text{ in} = \frac{5}{2} + \frac{27}{8} + \frac{39}{4} = \frac{20}{8} + \frac{27}{8} + \frac{78}{8} = \frac{125}{8} = 15\frac{5}{8}\text{ in.}$$

$$15\frac{5}{8}\text{ in} = 1\text{ ft } 3\frac{5}{8}\text{ in, so, all together, } 9\text{ ft} + 1\text{ ft } 3\frac{5}{8}\text{ in} = 10\text{ ft } 3\frac{5}{8}\text{ in.}$$

4. A 10-foot rope is to be cut into equal segments measuring 8 inches each. The total number of segments is

A. 1.
B. 8.
C. 15.
D. 20.
E. 40.

(C) The total number of inches in a 10-foot rope is $10 \times 12 = 120$ inches. The number of 8-inch segments that can be cut is $\frac{120}{8} = 15$.

5. A piece of wood measuring 16.5 inches long is cut into 2.75-inch pieces. How many smaller pieces of wood are there?

A. 3
B. 4
C. 5
D. 6
E. 7

(D) The number of smaller pieces is $\frac{16.5}{2.75} = 6$.

6. The least common multiple of 8, 12, and 20 is

A. 4.
B. 24.
C. 60.
D. 90.
E. 120.

(E) Factors of 8 are $2 \times 2 \times 2$; factors of 12 are $2 \times 2 \times 3$; factors of 20 are $2 \times 2 \times 5$. The least common multiple of 8, 12, and 20 is $2 \times 2 \times 2 \times 3 \times 5$ or 120.

7. A recipe calls for 3 cups of wheat and white flour combined. If $\frac{3}{8}$ of this is wheat flour, how many cups of white flour are needed?

A. $1\frac{1}{8}$
B. $1\frac{7}{8}$
C. $2\frac{3}{8}$
D. $2\frac{5}{8}$
E. $3\frac{3}{8}$

(B) If $\frac{3}{8}$ is wheat flour, then $1 - \frac{3}{8}$ or $\frac{5}{8}$ is white flour. So $3 \times \frac{5}{8} = \frac{15}{8} = 1\frac{7}{8}$ cups of white flour is needed.

8. Felix buys 3 books for $8.95 each. How much does he owe if he uses a $12.73 credit toward his purchase?

A. $39.58
B. $26.85
C. $21.68
D. $18.24
E. $14.12

(E) The total cost of the purchase is $\$8.95 \times 3 = \$26.85$. With a $12.73 credit, the amount owed is $\$26.85 - \$12.73 = \$14.12$.

9. 12 is 15% of what number?

A. 1.8
B. 8
C. 18
D. 36
E. 80

(E) Let n represent the number. If 12 is 15% of n, then $12 = 0.15n$. Divide both sides by 0.15. Therefore, $n = 80$.

10. How many distinct prime factors are there in 120?

A. 2
B. 3
C. 4
D. 5
E. 6

(B) Prime factors of 120 are $2 \times 2 \times 2 \times 3 \times 5$. Distinct factors are 2, 3, and 5. Therefore, there are 3 distinct prime factors.

11. The greatest common factor of 24 and 36 is

A. 6.
B. 8.
C. 12.
D. 36.
E. 60.

(C) Factors of 24 are $2 \times 2 \times 2 \times 3$. Factors of 36 are $2 \times 2 \times 3 \times 3$. The greatest common factor is $2 \times 2 \times 3 = 12$.

12. There are 800 employees at a company. If 60% drive to work and 30% take the train, how many employees arrive to work by car?

A. 240
B. 360
C. 480
D. 540
E. 600

(C) 60% arrive to work by car, so $800 \times 60\% = 480$.

13. Melodi eats $\frac{3}{8}$ of a pizza and divides the rest between her two friends. What percent of the pizza do her friends each receive?

A. 62.50%
B. 37.50%
C. 31.25%
D. 20.83%
E. 18.75%

(C) If $\frac{3}{8}$ of the pizza is eaten, then $1 - \frac{3}{8} = \frac{5}{8}$ remains. If that is divided by 2, then each receives $\frac{5}{8} \div 2 = \frac{5}{8} \times \frac{1}{2} = \frac{5}{16} = 0.3125 = 31.25\%$.

14. There are 72 freshmen in the band. If freshmen make up $\frac{1}{3}$ of the entire band, the total number of students in the band is

A. 24.
B. 72.
C. 144.
D. 216.
E. 288.

(D) Let n represent the number of students in the band. Then $\frac{1}{3}n = 72$, so $n = 72 \times 3 = 216$.

15. What percent of $\frac{3}{4}$ is $\frac{1}{8}$?

A. $9\frac{3}{8}\%$
B. 12%
C. $16\frac{2}{3}\%$
D. 20%
E. 60%

(C) Let p represent the unknown percent. Then $p \times \frac{3}{4} = \frac{1}{8}$. Solve for p by multiplying by the reciprocal of $\frac{3}{4} \cdot p \times \frac{3}{4} \times \frac{4}{3} = \frac{1}{8} \times \frac{4}{3} = \frac{4}{24} = \frac{1}{6}$. As a percent, $\frac{1}{6}$ is $\%\ 16\frac{2}{3}\%$.

Algebra

16. If $a = \frac{5}{2}$, then $\frac{1}{a}$ is

A. $\frac{2}{5}$.
B. 2.
C. $\frac{5}{2}$.
D. 5.
E. 10.

(A) Substitute $\frac{5}{2}$ for a. $\frac{1}{a} = \frac{1}{\frac{5}{2}} = 1 \div \frac{5}{2} = 1 \cdot \frac{2}{5} = \frac{2}{5}$.

17. Evaluate $3r^3 - 2s^2 + t$ if $r = -1$, $s = -2$, and $t = -3$.

A. –14
B. –8
C. 2
D. 4
E. 14

(C) Substituting the given values for r, s, and t into $3r^3 - 2s^2 + t$ gives $3(-1) - 2(-2) + (-3) = 3(-1) - 2(4) - 3 = -3 + 8 - 3 = 2$.

18. The scale on a map shows 500 feet for every $\frac{1}{4}$ inch. If two cities are 6 inches apart on the map, what is the actual distance they are apart?

A. 125 feet
B. 750 feet
C. 2,000 feet
D. 6,000 feet
E. 12,000 feet

(E) The proportion $\frac{500 \text{ ft}}{\frac{1}{4} \text{ in}} = \frac{x \text{ ft}}{6 \text{ in}}$ can be used to find the actual distance. Cross multiply. $500 \times 6 = \frac{1}{4}x$ so $3{,}000 = \frac{1}{4}x$ and $x = 3{,}000 \times 4 = 12{,}000$ ft.

19. $(3 - 1) \times 7 - 12 \div 2 =$

A. –2.
B. 1.
C. 2.
D. 4.
E. 8.

(E) Following the correct order of operations produces: $(3 - 1) \times 7 - 12 \div 2 = 2 \times 7 - (12 \div 2) = 14 - 6 = 8$.

20. Evaluate $3x + 7$ when $x = -3$.

A. –16
B. –2
C. 10
D. 16
E. 21

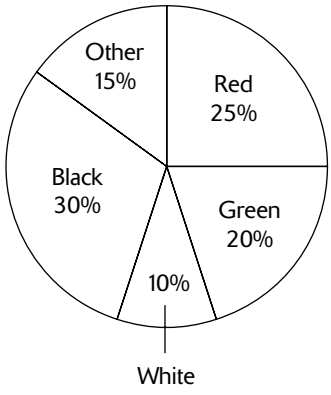
(B) Substitute –3 for x. Then $3(-3) + 7 = -9 + 7 = -2$.

21. Subtract $(2x^3 – 3x + 1) – (x^2 – 3x – 2)$.

A. $2x^2 + x + 1$
B. $2x^3 – x^2 – 6x – 1$
C. $x^3 – 6x – 1$
D. $x^2 + 3$
E. $3x^2 + x + 1$

(A) Subtraction can be changed to addition by changing the signs in the entire term being subtracted. $(2x^3 – 3x + 1) – (x^2 – 3x – 2) = (2x^3 – 3x + 1) + (x^2 – 3x – 2)$.

Combine like terms. $2x^3 – x^2 – 3x + 3x + 1 + 2 = 2x^3 – x^2 + 3$.



22. Heidi tallied the different car colors in the parking lot and summarized her results in a pie chart. There are 260 cars in the lot. How many cars are either red or black?

A. 65
B. 78
C. 117
D. 130
E. 143

(E) The percent of cars that are either red or black is 25% + 30% = 55%. The total cars that are either red or black is $260 \times 55\% = 143$.

23. Simplify $5(a – 2) – (4a – 6)$.

A. $9a – 4$
B. $a – 4$
C. $a – 8$
D. $a – 10$
E. $a + 4$

(B) $5(a – 2) – (4a – 6) = 5a – 10 – 4a + 6 = a – 4$.

24. The product of the square of x and three less than x is

A. $\sqrt{x}(x-3)$.
B. $\sqrt{x}(3-x)$.
C. $x^2(x-3)$.
D. $x^2(3-x)$.
E. $-x^2(x-3)$.

(C) The square of x is x^2. Three less than x is $x-3$. Their product is $x^2(x-3)$.

25. Factor $2a^2 - 4ab + ab - 2b^2$.

A. $(a+2b)(2a-b)$
B. $(a-2b)(2a+b)$
C. $(2a-b)(a+2b)$
D. $(2a+b)(a-b)$
E. $(a+2b)(a-2b)$

(B) Group the first two terms and the last two terms together: $(2a^2 - 4ab) + (ab - 2b^2)$. Factoring out common terms from each group gives $2a(a-2b)$ $b(a-2b)$. Common to both terms is $(a-2b)$. Factoring this out results in $(a-2b)(2a+b)$.

26. If b represents the cost of a book and m represents the cost of a magazine, which of the following expressions represents the cost of 5 books and 3 magazines if books cost twice as much as magazines?

A. $13m$
B. $8m$
C. $11b$
D. $8b$
E. $13b$

(A) If books are twice as much as magazines, then $b = 2m$. 5 books + 3 magazines = $5b + 3m$. Substituting $2m$ for b gives $5(2m) + 3m = 10m + 3m = 13m$.

27. Rachel ran $\frac{1}{2}$ mile in 4 minutes. At this rate, how many miles can she run in 15 minutes?

A. $1\frac{7}{8}$
B. $2\frac{1}{8}$
C. 4
D. 30
E. 60

(A) The proportion $\frac{\frac{1}{2}\text{ mile}}{4\text{ minutes}} = \frac{x\text{ miles}}{15\text{ minutes}}$ models this situation. Cross multiply. $\frac{1}{2} \times 15 = 4x$ so $\frac{15}{2} = 4x$ and $x = \frac{15}{2} \cdot \frac{1}{4} = \frac{15}{8} = 1\frac{7}{8}$ miles.

28. What is the value of $(-8)^{\frac{2}{3}}$?

A. –4
B. –2
C. 2
D. 4
E. 16

(D) $(-8)^{\frac{2}{3}} = \left(\sqrt[3]{-8}\right)^2 = (-2)^2 = 4$.

29. If $2^{b+3} = \frac{1}{8}$, then b =

A. –6.
B. –5.
C. –3.
D. 0.
E. 2.

(A) $\frac{1}{8} = \frac{1}{2^3} = 2^{-3}$ so $2^{b+3} = 2^{-3}$ and $b + 3 = -3$. Therefore, $b + 3 - 3 = -3 - 3 = -6$.

30. Three boxes are needed to hold 18 reams of paper. How many boxes are needed for 90 reams?

A. 5
B. 6
C. 9
D. 15
E. 30

(D) The proportion $\frac{3 \text{ boxes}}{18 \text{ reams}} = \frac{x \text{ boxes}}{90 \text{ reams}}$ can be used to find the number of boxes. Cross multiply. $3 \times 90 = 18x$ so $270 = 18x$ and $x = \frac{270}{18} = 15$ boxes.

31. Multiply $(5a^3bc^2)(-3a^2c)$.

A. $-15a^5bc^3$
B. $15a^5bc^3$
C. $-15a^6bc^2$
D. $2abc$
E. $2a^5bc^3$

(A) $(5a^3bc^2)(-3a^2c) = 5 \cdot -3\ a^{3+2}bc^{2+1} = -15a^5bc^3$.

32. The cube root of 512 is

A. 6.
B. 8.
C. 56.
D. $170\frac{2}{3}$.
E. 1,536.

(B) The cube root of 512 is $\sqrt[3]{512} = \sqrt[3]{8 \times 8 \times 8} = 8$.

33. Simplify $\left(\frac{a^{-3}b^{2}}{2ab^{-1}}\right)^{-3}$.

A. $\frac{2a^{6}}{b}$

B. $\frac{8a^{12}}{b^{9}}$

C. $\frac{a^{8}}{8b^{3}}$

D. $\frac{a^{12}}{8b^{9}}$

E. $\frac{8a^{8}}{b}$

(B) $\left(\frac{a^{-3}b^{2}}{2ab^{-1}}\right) = \frac{a^{9}b^{-6}}{2^{-3}a^{-3}b^{3}} = 2^{3}a^{9-(-3)}b^{-6-3} = 8a^{12}b^{-9} = \frac{8a^{12}}{b^{9}}$.

34. Simplify $\frac{9x^{2}y^{3}z - 12xy^{2}z^{2}}{3yz}$.

A. $3xy^2z - 4xyz$

B. $3x^2y^2 - 12xyz$

C. $3x^2y^2 - 4xyz$

D. $3y^2 - 4xy^2z^2$

E. $9x^2y^3z - 4xyz$

(C) $\frac{9x^{2}y^{3}z - 12xy^{2}z^{2}}{3yz} = \frac{9x^{2}y^{3}z}{3yz} - \frac{12xy^{2}z^{2}}{3yz} = 3x^{2}y^{2} - 4xyz.$

35. What is the value of $\left(\frac{9}{4}\right)^{-\frac{1}{2}}$?

A. $-\frac{2}{3}$

B. $-\frac{16}{81}$

C. $\frac{16}{81}$

D. $\frac{2}{3}$

E. $\frac{3}{2}$

(D) $\left(\frac{9}{4}\right)^{-\frac{1}{2}} = \left(\frac{4}{9}\right)^{\frac{1}{2}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$

36. What is the value of $-27^{\frac{2}{3}}$?

A. –9
B. –6
C. –3
D. 3
E. 9

(A) $-27^{\frac{2}{3}} = -\left(27^{\frac{2}{3}}\right) = -\left(\sqrt[3]{27}\right)^2 = -(3)^2 = -9$.

37. If $w - 3 = 3 - w$, what is the value of w^2?

A. 0
B. 1
C. 3
D. 6
E. 9

(E) Solve for w by adding w to both sides. $w - 3 + w = 3 - w + w$ so $2w - 3 = 3$. Adding 3 to both sides gives $2w = 6$. So $\frac{2w}{2} = \frac{6}{2}$ and $w = 3$. Therefore, $w^2 = 3^2 = 9$.

38. If $\frac{m}{n} = \frac{3}{5}$, what is the value of $m + n$?

A. 2
B. 8
C. $\frac{6}{5}$
D. $\frac{9}{25}$
E. It cannot be determined.

(E) The values of m and n can be anything as long as they are in the proportion $\frac{m}{n} = \frac{3}{5}$. For example, it could be that $m = 3$ and $n = 5$. However, it is also possible that $m = 6$ and $n = 10$. Therefore, it is not possible to determine the sum.

39. Simplify $\frac{x^2 - 25}{5 - x}$.

A. $x + 5$
B. $x - 5$
C. $-(x + 5)$
D. $5 - x$
E. $x - 20$

(C) $\frac{x^2 - 25}{5 - x} = \frac{(x+5)(x-5)}{5 - x} = \frac{(x+5)(x-5)}{-(x-5)} = \frac{(x+5)}{-1} = -(x+5)$.

40. If $3^{4x+2} = 9^{x-1}$, then what is the value of x?

A. –2
B. –1
C. 0
D. 1
E. 2

(A) Begin by rewriting 9 as 3^2, so that $3^{4x+2} = 9^{x-1}$ becomes $3^{4x+2} = 3^{2(x-1)}$. The only way this can be true is if $4x + 2 = 2(x - 1)$. This equation is true if $x = -2$.

Geometry

41. Which expression represents the volume of a cylinder whose height is equivalent to the length of the radius?

A. πr^2
B. πr^3
C. $(\pi r)^2$
D. $(\pi r)^3$
E. $\frac{\pi r^3}{2}$

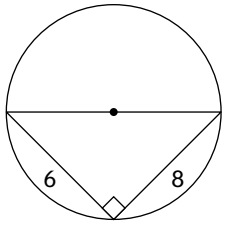
(B) The volume of a cylinder is given by the formula $V = \pi r^2 h$, where r is the radius of the circular base and h is the height. Since $h = r$, $V = \pi r^2 r = \pi r^3$.

42. The length of a rectangle is three times its width. If the perimeter of the rectangle is 48, what is its area?

A. 144
B. 108
C. 96
D. 54
E. 48

(D) The perimeter of a rectangle is $l + w + l + w = 48$. Since $l = 3w$, the perimeter is $3w + w + 3w + w = 48$, so $8w = 48$ and $w = 6$. Therefore, the length is 3×6 or 18, and the area of the rectangle is $l \times w = 18 \times 3 = 54$.

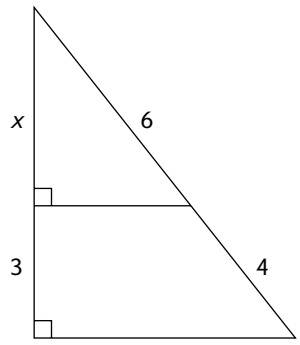
43. Find the length of the radius in the following figure.



A. 3
B. 4
C. 5
D. 8
E. 10

(C) The hypotenuse of the triangle is the diameter of the circle. By the Pythagorean Theorem, $d^2 = 6^2 + 8^2 = 36 + 64 + 100$. So $d = \sqrt{100} = 10$, and the radius is $\frac{10}{2} = 5$.

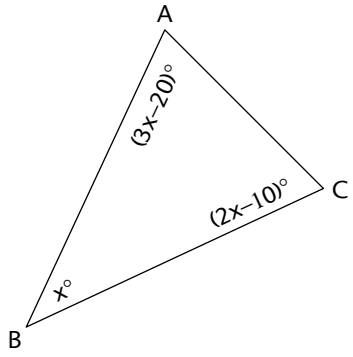
44. Find the value of x in the figure:



A. 4.5
B. 4.6
C. 4.8
D. 5
E. 5.2

(A) The proportion $\frac{x}{6} = \frac{x+3}{10}$ can be used to find x. Cross multiply. $10x = 6(x + 3)$ and $10x = 6x + 18$. Bring all x terms to one side by subtracting $6x$ from each side. Then, $4x = 18$ and $x = \frac{18}{4} = 4.5$.

45. What is the measure of $\angle A$?

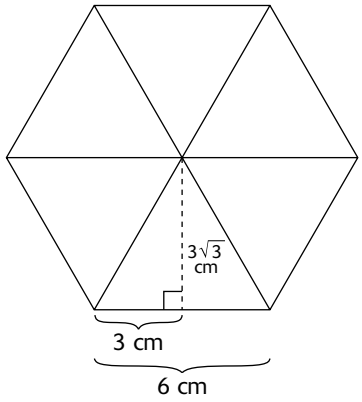


A. 35°
B. 60°
C. 65°
D. 75°
E. 85°

(E) The sum of all angles in a triangle equal 180°. So $(3x – 20)° + x° + (2x – 10)° = 180°$. $3x + x + 2x – 20 – 10 = 180$ and $6x – 30 = 180$. Then $6x = 210$ and $x = \frac{210}{6} = 35$. Therefore, $\angle A$ is $3(35) – 20$ or 85°.

46. Find the area of a regular hexagon whose sides measure 6 cm.

A. $9\sqrt{2}$
B. 36
C. $54\sqrt{3}$
D. 108
E. 216



(C) A regular hexagon is made up of six equilateral triangles. Find the area of one equilateral triangle and multiply that by 6 to find the area of the hexagon. The height, or altitude, of a triangle can be found by the Pythagorean Theorem. The right triangle formed by the altitude has a hypotenuse of 6 and a shorter leg of $\frac{6}{2}$ or 3. So $3^2 + h^2 = 6^2$ so $9 = h^2 = 36$ and $h^2 = 27$. Therefore, $h = \sqrt{27} = 3\sqrt{3}$. The area of one equilateral triangle is $\frac{1}{2}bh = \frac{1}{2} \cdot 6 \cdot 3\sqrt{3} = 9\sqrt{3}$, and the area of the hexagon is $6 \cdot 9\sqrt{3} = 54\sqrt{3}$.

47. What is the slope of the line $x = -3y + 9$?

A. –3
B. $-\frac{1}{3}$
C. $\frac{1}{3}$
D. 3
E. 9

(B) Begin by rewriting $x = -3y + 9$ in slope-intercept form as $y = -\frac{1}{3}x + 3$. The slope of the line is the coefficient of x, that is, $-\frac{1}{3}$.

48. The angles of a triangle are in the ratio 3:4:5. What is the measure of the smallest angle?

A. 15°
B. 30°
C. 45°
D. 60°
E. 75°

(C) Angles in a triangle add to 180°. So $3x + 4x + 5x = 180°$ and $12x = 180°$. Dividing both sides by 12 results in $x = 15°$. The smallest angle is represented by $3x = 3(15°) = 45°$.

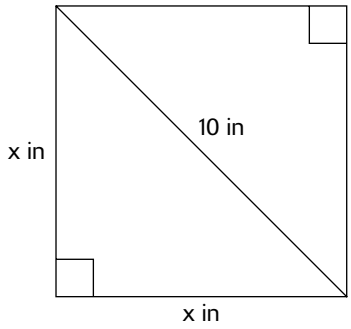
49. A cardboard box has a length of 3 feet, height of $2\frac{1}{2}$ feet, and a depth of 2 feet. If the length and depth are doubled, by what percent does the volume of the box change?

A. 200%
B. 300%
C. 400%
D. 500%
E. 600%

(B) The volume of the original box is $3 \times 2\frac{1}{2} \times 2 = 15$. The volume of the box with the length and depth doubled is $6 \times 2\frac{1}{2} \times 4 = 60$. The amount of change in volume is 60 – 15 = 45. The percent change is the amount of change in volume divided by the original volume. $\frac{45}{15} = 3 = 300\%$.

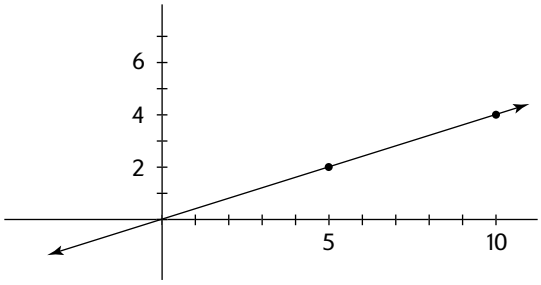
50. The diagonal of a square is 10 inches. What is the area of the square?

A. 40 in^2
B. 50 in^2
C. 80 in^2
D. 100 in^2
E. 150 in^2



(B) Let x represent a side of the square. The area of the square is x^2. To find the value of x^2, use the Pythagorean Theorem. $x^2 + x^2 = 10^2$ so $2x^2 = 100$ and $x^2 = \frac{100}{2}$ or 50 in^2.

51. The slope of the line shown is



A. $-\frac{2}{5}$.

B. $-\frac{5}{2}$.

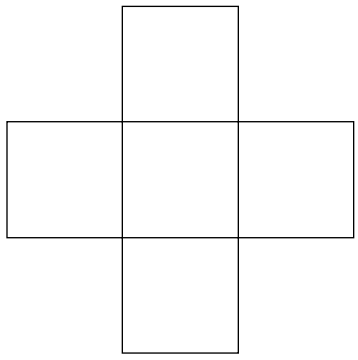
C. $\frac{2}{5}$.

D. $\frac{2}{3}$.

E. $\frac{5}{2}$.

(C) Slope is found by identifying two points on the line and finding the $\frac{\text{change in } y}{\text{change in } x}$. The points (0, 0) and (5, 2) form the slope $\frac{2-0}{5-0} = \frac{2}{5}$.

52. The figure contains 5 equal squares. If the area is 405, what is the perimeter?



A. 81
B. 90
C. 108
D. 144
E. 216

(C) The area of one square is $\frac{405}{5} = 81$. So the length of each side is $\sqrt{81} = 9$. The total number of sides in the figure is 12, so the perimeter is $9 \times 12 = 108$.

53. What is the diameter of a circle whose circumference is equivalent to its area?

A. 1
B. 2
C. 3
D. 4
E. 6

(D) The circumference of a circle is given by the formula $C = 2\pi r$, and the area of a circle is given by $A = \pi r^2$. If the circumference is equal to the area, then $2\pi r = \pi r^2$. Solving for r, $\frac{2\pi r}{\pi r} = \frac{\pi r^2}{\pi r}$ and $2 = r$. The diameter is $2r$, or 4.

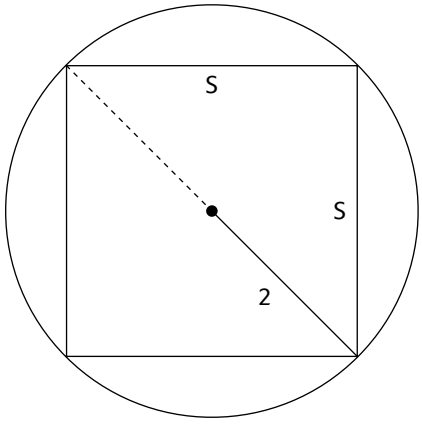
54. A line parallel to the line $y = 5x + 9$ has a slope of

A. –5.
B. $-\frac{1}{5}$.
C. $\frac{1}{5}$.
D. $\frac{9}{5}$.
E. 5.

(E) Parallel lines have the same slope. Since the given line is in the slope-intercept form, the slope is the coefficient of the x term, which is 5. This must also be the slope of a parallel line.

55. A square garden is to be built inside a circular area. Each corner of the square touches the circle. If the radius of the circle is 2, how much greater is the area of the circle than the square?

A. $4 - 4\pi$
B. $4 - 8\pi$
C. $4\pi - 4$
D. $4\pi - 8$
E. $8\pi - 8$



(D) Find the difference between the area of the circle and the area of the square. The area of the circle is $\pi r^2 = \pi \cdot 2^2 = 4\pi$. The area of the square is s^2, where s represents the length of the square. The radius is half the length of the square's diagonal, so the diagonal is 4. By the Pythagorean Theorem, $s^2 + s^2 = 4^2$. $2s^2 = 16$, so $s^2 = 8$. The difference in area is $4\pi - 8$.

56. What is the slope of the line $2x + y = 7$?

A. –2
B. 1
C. 2
D. $\frac{7}{2}$
E. 7

(A) Re-write the equation in slope-intercept form as $y = –2x + 7$. The slope is the coefficient of x, which is –2.

57. The volume of a cube is 343 cm^3. The surface area of the cube is

A. 7 cm^2.
B. 49 cm^2.
C. 294 cm^2.
D. 588 cm^2.
E. 2401 cm^2.

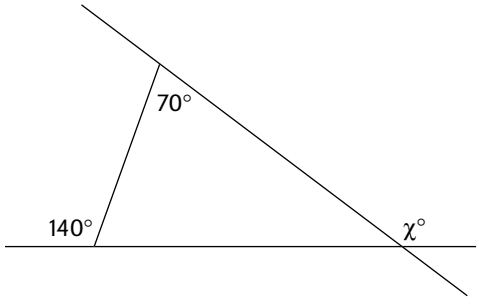
(C) The volume of a cube is s^3, where s represents the length of an edge. Surface area is $6s^2$. If the volume = 343 cm^3, then $s = \sqrt[3]{343} = \sqrt[3]{7 \cdot 7 \cdot 7} = 7$. So the surface area is $6 \cdot 7^2 = 294$ cm^2.

58. A cylinder whose height is 8 inches has a volume of $128\pi cm^3$. If the radius is doubled and its height is cut in half, the volume of the resulting cylinder is

A. 64π cm^3.
B. 128π cm^3.
C. 256π cm^3.
D. 512π cm^3.
E. $1{,}024\pi$ cm^2.

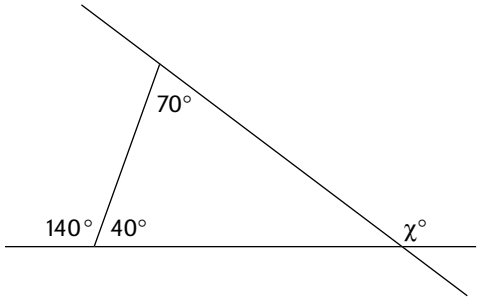
(C) The volume of a cylinder is $\pi r^2 h$. In the original cylinder, $\pi r^2 8 = 128\pi$, so $r^2 = \frac{128\pi}{8\pi} = 16$, and the radius, r, equals $\sqrt{16} = 4$. In the new cylinder, the radius is doubled to 8, and the height is cut in half to 4. The resulting volume is $\pi \cdot 8^2 \cdot 4 = 256\pi$ cm^3.

59. The value of x is



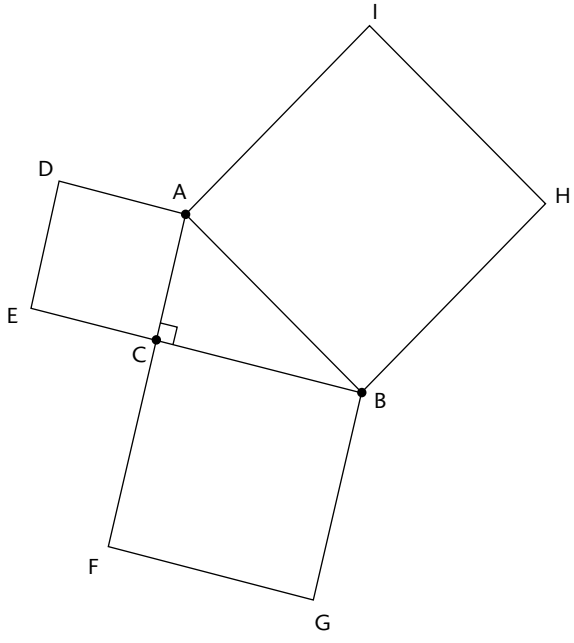
A. 30°.
B. 70°.
C. 110°.
D. 140°.
E. 180°.

(C)



The angle adjacent to the 140° angle is 40° since supplementary angles add to 180°. The angles of a triangle add to 180°, so the angle adjacent to angle *x* is 180° – 70° – 40° = 70°. Angle *x* and 70° are supplementary, so *x* = 180° – 70° = 110°.

60. Squares ADEC, BCFG, and ABHI are shown. If the area of ADEC is 81 and the area of BCFG is 144, what is the perimeter of △ABC?



A. 36
B. 54
C. 72
D. 225
E. 450

(A) Since the area of ADEC is 81, AC = $\sqrt{81}$= 9. Since the area of BCFG is 144, BC = $\sqrt{144}$ = 12. Use the Pythagorean Theorem to find the length of the remaining side AB. $AB^2 + 12^2$, so $AB^2 = 81 + 144 = 225$ and AB = $\sqrt{225}$ = 15. Therefore, the perimeter of the triangle = 9 + 12 + 15 = 36.

Conversions

61. Yan can read 2 pages in 3 minutes. At this rate, how long will it take him to read a 360 page book?

A. 30 minutes
B. 2 hours
C. 6 hours
D. 9 hours
E. 12 hours

(D) Using the ratio $\frac{\text{pages}}{\text{minutes}}$, the proportion $\frac{2}{3} = \frac{360}{x}$ can be used to find the time. Cross multiply. $2x = 3 \times 360$, so $2x = 1080$ and $x = \frac{1080}{2} = 540$ minutes. Convert minutes to hours. There are 60 minutes in 1 hour, so $\frac{540}{60} = 9$ hours.

62. How many omelets can be made from 2 dozen eggs if an omelet contains 3 eggs?

A. 1
B. 3
C. 4
D. 6
E. 8

(E) There are 24 eggs in 2 dozen eggs. If 3 eggs are in an omelet, then 24 ÷ 3, or 8 omelets can be made.

63. A blueprint has a scale of 3 feet per $\frac{1}{2}$ inch. If a bathroom is $1\frac{1}{2}$ inches × 2 inches, what are its actual dimensions?

A. $4\frac{1}{2}$ feet × 6 feet
B. 6 feet × $7\frac{1}{2}$ feet
C. $7\frac{1}{2}$ feet × 9 feet
D. 6 feet × 9 feet
E. 9 feet × 12 feet

(E) If the blueprint shows $\frac{1}{2}$ inch for every 3 feet, then 1 inch represents 6 feet. The actual dimensions of a room $1\frac{1}{2}$ inches × 2 inches would be $(1\frac{1}{2} \times 6)$ by (2×6) or 9 feet by 12 feet.

64. Tiling costs $2.89 per square foot. What is the cost to tile a kitchen whose dimensions are 4 yards by 5 yards?

A. $57.80
B. $62.28
C. $173.40
D. $289.00
E. $520.20

(E) There are 3 feet in a yard, so a kitchen 4 yards by 5 yards is equivalent to (4×3) feet by (5×3) feet, or 12 feet by 15 feet. The area of the kitchen is $12 \times 15 = 180$ square feet. The cost to tile is $\$2.89 \times 180 = \$520.20$.

65. A machine can produce 8,000 widgets in 3 hours. How many widgets are produced in one day?

A. 96,000
B. 64,000
C. 48,000
D. 32,000
E. 8,000

(B) If a machine produces 8,000 widgets in 3 hours, it produces $\frac{8000}{3}$ widgets in one hour. There are 24 hours in a day, so $\frac{8000}{3} \times 24$ or 64,000 widgets are produced in one day.

66. Dennis ran a race in 2.2 minutes. Kayla ran the same race in 124 seconds. What is the difference between these two times?

A. 2 seconds
B. 8 seconds
C. 12 seconds
D. 14 seconds
E. 22 seconds

(B) Convert 2.2 minutes to seconds. $2.2 \times 60 = 132$ seconds. The difference in the two times is $132 - 124 = 8$ seconds.

67. Stanley can type 35 words per minute. If it takes him a half hour to type a document, about how many words are in the document?

A. 525
B. 900
C. 1,050
D. 1,500
E. 2,100

(C) There are 30 minutes in a half hour. $30 \times 35 = 1{,}050$ words.

68. Floor tiling costs $13.50 per square yard. What would it cost to tile a room 15 feet long by 18 feet wide?

A. $20
B. $405
C. $425
D. $1,350
E. $3,645

(B) The area of a room 15 feet wide by 18 feet long is $15 \times 18 = 270$ square feet. Since there are 3 feet in a yard, there are 3×3 or 9 feet in a square yard. Convert 270 square feet to square yards. $\frac{270}{9} = 30$ square yards. Since the cost is $13.50 per square yard, the total cost is $\$13.50 \times 30$ or $405.

69. A car travels 20 miles in 30 minutes. At this rate, how far will the car travel in 2 hours?

A. 40 miles
B. 60 miles
C. 80 miles
D. 100 miles
E. 120 miles

(C) There are 120 minutes in 2 hours. Setting up a proportion yields $\frac{20 \text{ miles}}{30 \text{ minutes}} = \frac{x \text{ miles}}{120 \text{ minutes}}$. Cross multiplying results in $30x = 20 \times 120$ or $30x = 2400$. Dividing both sides by 30 gives $x = \frac{2400}{30} = 80$ miles.

70. How many blocks 6" × 4" × 4" can fit in a box 8' × 6' × 4'?

A. 2
B. 48
C. 288
D. 576
E. 3,456

(E) Convert the dimensions of the box from feet to inches. 8' × 6' × 4' is equivalent to (8 × 12 in) × (6 × 12 in) × (4 × 12 in) = 96 in × 72 in × 48 in. The volume = 96 × 72 × 48 = 331,776. The volume of each block is 6 × 4 × 4 = 96. The number of blocks that fit in the box is $\frac{331,776}{96} = 3,456$.

Trigonometry and Applied Mathworks

71. If sin $a > 0$ and cos $a < 0$, then $-a$ must lie in which quadrant?

A. I
B. II
C. III
D. IV
E. Either I or II

(B) The sine function is positive in the first and second quadrants. The cosine function is negative in the second and third quadrants. Overall, then, $-a$ must lie in the second quadrant.

72. Which of the following values of x is a solution of the equation cos $x = -1$?

A. $x = 0°$
B. $x = 45°$
C. $x = 90°$
D. $x = 180°$
E. $x = 270°$

(D) The only one of the given angles for which cos $x = -1$ is $x = 180°$.

73. The expression $\tan\theta \cos\theta \csc\theta$ is equivalent to

A. 1.
B. sin θ.
C. tan θ.
D. sec θ.
E. cos θ.

(A) $\tan\theta \cos\theta \csc\theta = \frac{\sin\theta}{\cos\theta} \times \cos\theta \times \frac{1}{\sin\theta} = 1$.

74. What is the period of the function $g(x) = 6\sin 2x$?

A. 2
B. π
C. 4
D. 2π
E. 6π

(B) To find the period, set $2x$ equal to 2π, and solve for x. It is easy to see that $x = \pi$ is the period.

75. If $\sin b < 0$ and $\cos b < 0$, then $\angle b$ must lie in which quadrant?

A. I
B. II
C. III
D. IV
E. Either III or IV

(C) The sine function is negative in quadrants III and IV, while the cosine function is negative in quadrants II and III. Therefore, they are both negative in the third quadrant.

76. For which of the following values of x is the function $h(x) = \tan 2x$ undefined?

A. 0
B. $\frac{\pi}{4}$
C. $\frac{\pi}{2}$
D. π
E. 2π

(B) A good way to think about this problem is to recall that $\tan 2x = \frac{\sin 2x}{\cos 2x}$. Therefore, $\tan 2x$ will be undefined when $\cos 2x = 0$. Note that $\cos\left(\frac{\pi}{2}\right) = 0$, and, at $\frac{\pi}{4}$, $\cos 2x = \cos\left(\frac{\pi}{2}\right) = 0$. Thus, $\tan 2x$ is undefined at $\frac{\pi}{4}$.

77. The expression $\tan\theta \cos\theta \sec\theta$ is equivalent to

A. $\cot\theta$.
B. $\sec\theta$.
C. $\sin\theta$.
D. $\tan\theta$.
E. $\cos\theta$.

(D) $\tan\theta \cos\theta \sec\theta = \frac{\sin\theta}{\cos\theta} \times \cos\theta \times \frac{1}{\cos\theta} = \tan\theta$.

78. Which of the following values of x is a solution to the equation $\csc x = 2$?

A. $x = 0°$
B. $x = 30°$
C. $x = 45°$
D. $x = 60°$
E. $x = 90°$

(B) Recall that the cosecant function is the reciprocal of the sine function. Thus, csc $x = 2$ is the same as sin $x = \frac{1}{2}$, which is true when $x = 30°$.

79. The expression $\sec^2\theta - 1$ is equivalent to

A. $\tan^2\theta$.
B. $\csc^2\theta$.
C. $\sin^2\theta$.
D. $\cos^2\theta$.
E. $\sec^2\theta$.

(A) One of the Pythagorean identities in trigonometry is $\tan^2\theta + 1 = \sec^2\theta$. Thus, $\tan^2\theta = \sec^2\theta. - 1$.

80. If $\csc\theta > 0$ and $\cos\theta < 0$, then what quadrant is the angle θ in?

A. I
B. II
C. III
D. IV
E. Either I or II

(B) The cosecant function is positive in quadrants I and II. The cosine function is negative in quadrants II and III. Thus, the angle must be in the second quadrant.

81. Rae earns \$8.40 an hour plus an overtime rate equal to $1\frac{1}{2}$ times her regular pay for each hour worked beyond 40 hours. What are her total earnings for a 45 hour work week?

A. \$336
B. \$341
C. \$370
D. \$399
E. \$567

(D) The overtime rate is $\$8.40 \times 1.5 = \$12.60$. Five hours of overtime were completed, so the total earnings are $(\$8.40 \times 40) + (\$12.60 \times 5) = \$336 + \$63 = \$399$.

82. Davis donates $\frac{4}{13}$ of his paycheck to his favorite charity. If he donates \$26.80, what is the amount of his paycheck?

A. \$8.25
B. \$82.50
C. \$87.10
D. \$92.25
E. \$348.40

(C) Let p represent the amount of the paycheck. $\frac{4}{13}p = \$26.80$, so $p = \$26.80 \cdot \frac{13}{4} = \$87.10$.

83. One phone plan charges a $20 monthly fee and $0.08 per minute on every phone call made. Another phone plan charges a $12 monthly fee and $0.12 per minute for each call. After how many minutes would the charge be the same for both plans?

A. 60 minutes
B. 90 minutes
C. 120 minutes
D. 180 minutes
E. 200 minutes

(E) Let m represent the minutes of the phone calls. The monthly charge for the first plan is $20 + 0.08m$. The monthly charge for the second plan is $12 + 0.12m$. When the monthly charges are the same, $20 + 0.08m = 12 + 0.12m$. Solve for m to find the number of minutes both plans have the same rate.

$20 + 0.08m - 0.08m = 12 + 0.12m - 0.08m$

$20 = 12 + 0.04m$

$20 - 12 = 12 + 0.04m - 12$

$8 = 0.04m$ so $m = \frac{8}{0.04} = \frac{800}{4} = 200$ minutes.

84. A sweater originally priced at $40 is on sale for $30. What percent has the sweater been discounted?

A. 20%
B. 25%
C. 33%
D. 70%
E. 75%

(B) The amount of discount is $40 – $30 = $10. The percent of discount is the amount of discount divided by the original price. $\frac{10}{40} = \frac{1}{4} = 25\%$.

85. Staci earns $9.50 an hour plus 3% commission on all sales made. If her total sales during a 30-hour work week were $500, how much did she earn?

A. $15
B. $250
C. $285
D. $300
E. $435

(D) For a 30-hour week with $500 in sales, total earnings are $(30 \times \$9.50) + (3\% \times \$500) = \$285 + \$15 = \$300$.

86. One-fourth of the cars purchased at a dealership are luxury models. If 360 luxury models were purchased last year, how many total cars were purchased?

A. 90
B. 250
C. 1,440
D. 2,880
E. 3,600

(C) $\frac{1}{4}$ of the total cars, t, sold are luxury. Luxury cars sold = 360, so $\frac{1}{4}t = 360$ and $t = 360, \times 4 = 1{,}440$ total cars sold.

87. A television is on sale for 20% off. If the sale price is $800, what was the original price?

A. $160
B. $640
C. $960
D. $980
E. $1,000

(E) If an item is discount 20%, the sale price is 80% of the original price. Let p represent the original price. Then $\$800 = 80\% \times p$ and $p = \frac{800}{80\%} = \frac{800}{.80} = \$1{,}000$.

88. A barrel holds 60 gallons of water. If a crack in the barrel causes $\frac{1}{2}$ a gallon to leak out each day, how many gallons of water remain after 2 weeks?

A. 30
B. 53
C. $56\frac{1}{2}$
D. 58
E. 59

(B) In 2 weeks, or 14 days, $\frac{1}{2} \times 14 = 7$ gallons leak out, leaving $60 - 7 = 53$ gallons.

89. A restaurant bill without tax and tip comes to $38.40. If a 15% tip is included after a 6% tax is added to the amount, how much is the tip?

A. $6.11
B. $6.05
C. $5.76
D. $5.15
E. $2.30

(A) The tax on the bill is $\$38.40 \times 6\% = \$2.30$. The amount, including tax, is $\$38.40 + \$2.30 = \$40.70$. The tip is $\$40.70 \times 15\% = \$6.11$.

90. A savings account earns $2\frac{1}{4}\%$ interest each year. How much interest is earned on a $1,000 deposit after a 5-year period?

A. $22.50
B. $100.00
C. $112.50
D. $124.00
E. $150.00

(C) Interest = Principle × Rate × Time. Thus, Interest = $\$1{,}000 \times 2\frac{1}{4}\% \times 5 = \$1{,}000 \times 0.0225 \times 5 = \$112.50$.

Probability and Statistics

91. Kyle ran 3 miles in $17\frac{1}{2}$ minutes on Saturday, $4\frac{1}{2}$ miles in 22 minutes on Sunday, and 2 miles in 9 minutes on Monday. What was Kyle's average rate of speed while running?

A. 1.6 minutes per mile
B. 5.1 minutes per mile
C. 5.6 minutes per mile
D. 16.2 minutes per mile
E. 17.8 minutes per mile

(B) Average is the total time divided by the total miles run. The total time is 17.5 + 22 + 9 = 48.5 minutes. The total number of miles run is 3 + 4.5 + 2 = 9.5. The average is $\frac{48.5}{9.5}$ = 5.1 minutes per mile.

92. Tanya's bowling scores this week were 112, 156, 179, and 165. Last week, her average score was 140. How many points did her average improve?

A. 18
B. 13
C. 11
D. 10
E. 8

(B) The average is found by adding up all the scores and dividing by the total number of scores. The average this week is $\frac{112 + 156 + 179 + 165}{4} = \frac{612}{4}$ = 153. The amount of improvement is 153 – 140 = 13.

93. For which of the following sets of numbers is the median the same as the arithmetic mean?

A. {–2, –1, 0, 2, 3}
B. {0, 2, 3, 4, 5}
C. {–1, 1, 4}
D. {0, 2, 4, 6, 10}
E. {–2, –1, 0, 1, 2}

(E) The median of an odd amount of numbers is the value in the middle when the numbers are put in numerical order. Note, then, that the median of this set is 0, which is also the mean.

94. What is the probability of rolling a sum of 9 using two dice?

A. $\frac{1}{9}$
B. $\frac{1}{6}$
C. $\frac{7}{36}$
D. $\frac{1}{4}$
E. $\frac{5}{12}$

(A) There are 4 possible ways to roll a 9 using 2 dice: 3 and 6, 4 and 5, 5 and 4, 6 and 3. The total number of possible outcomes when rolling 2 dice is 6^2 or 36. Therefore, the probability of rolling a 9 is $\frac{4}{36} = \frac{1}{9}$.

95. Two runners finished a race in 80 seconds; another runner finished the race in 72 seconds; and the final runner finished in 68 seconds. The average of these times is

A. 73 seconds.
B. 74 seconds.
C. 75 seconds.
D. 76 seconds.
E. 77 seconds.

(C) Since two runners finished in 80 seconds, the average of 80, 80, 72, and 68 must be found. This average is $\frac{80 + 80 + 72 + 68}{4} = \frac{300}{4} = 75$ seconds.

96. What is the mode of the set of numbers {1, 1, 1, 2, 2, 3, 3, 3, 3, 4, 4, 4, 5, 6, 6, 6}?

A. 1
B. 3
C. 4
D. 5
E. 6

(B) The mode is the number that occurs the most often, which is, in the this case, 3.

97. In a standard deck of playing cards, a king of hearts is drawn and not replaced. What is the probability of drawing another king from the deck?

A. $\frac{3}{52}$
B. $\frac{1}{17}$
C. $\frac{1}{13}$
D. $\frac{1}{4}$
E. $\frac{3}{13}$

(B) Probability is $\frac{\text{number of expected outcomes}}{\text{number of possible outcomes}}$. Since one king was drawn and not replaced, three kings remain in the deck of 51 cards. So the probability of drawing another king is $\frac{3}{51} = \frac{1}{17}$.

98. If a 4-digit number is formed at random from the digits 3, 5, 7, and 8, and each digit is used once, what is the probability that the number is even?

A. $\frac{1}{16}$
B. $\frac{1}{8}$
C. $\frac{1}{4}$
D. $\frac{1}{3}$
E. $\frac{1}{2}$

(C) The only even digit is 8, so for the 4-digit number to be even, 8 must be the last digit. If the number is formed at random, there is a one in four chance that the last digit will be 8.

99. What is the probability of flipping 3 heads in a row using a fair coin?

A. $\frac{1}{8}$

B. $\frac{1}{4}$

C. $\frac{3}{8}$

D. $\frac{1}{2}$

E. $\frac{2}{3}$

(A) The probability of flipping one head is $\frac{1}{2}$. The probability of flipping three heads in a row is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$ or $\frac{1}{8}$.

100. A 4-digit number is formed at random from the digits 1, 2, 3, and 4. If digits *can* be repeated in the number, what is the probability that the number is greater than 2,500?

A. $\frac{1}{16}$

B. $\frac{1}{8}$

C. $\frac{1}{4}$

D. $\frac{1}{3}$

E. $\frac{1}{2}$

(**E**) The number will be larger than 2,500 if the first digit is either 3 or 4. Since all digits are selected at random, the probability that a 3 or a 4 is selected for the first digit is $\frac{1}{2}$.