

# SAT MATH PRACTICE TEST 16

Before we begin, note that the SAT will change to an online format beginning in fall of 2023. The Math section will be divided into four sections:

- Algebra
- Advanced Math
- Problem Solving and Data Analysis
- Geometry and Trigonometry

## Algebra

The Algebra portion of the digital SAT Math section will require the test taker to be proficient in the following areas:

- Linear equations in one and two variables
- Linear functions
- Systems of equations
- Inequalities in one or two variables

## Solving Equations

### Drill 1

For multiple-choice questions, solve the problem and pick the correct answer from the provided choices. Each multiple-choice question has only one correct answer. For student-produced response questions, solve each problem and write down your answers separately.

$$2(-3x - 7) = 4 - 2x$$

1. In the equation above, what is the value of  $x$ ?

- (A)  $-9$
- (B)  $\frac{-9}{2}$
- (C)  $\frac{2}{9}$
- (D)  $\frac{7}{2}$

2. If  $\frac{3x}{5} = 12$ , what is the value of  $x$ ?

- (A) 15
- (B) 20
- (C) 30
- (D) 60

### TIP

In order to excel in the Algebra section, the test taker should be well-versed in the order of operations. Use the term PEMDAS to remember the sequence of mathematical operations in any math question.

**P:** Parentheses

**E:** Exponents

**M:** Multiplication

**D:** Division

**A:** Addition

**S:** Subtraction

Multiplication and division are to be treated with equal priority, as are addition and subtraction. If a line of mathematical calculation contains solely multiplication and division or solely addition and subtraction, calculate from left to right.

3. If  $\frac{x+1}{4} = y$  and  $y = 4$ , what is the value of  $x$ ?

- (A) 15
- (B) 13
- (C) 11
- (D) 8

$$|m - 3| = 12$$

$$|n + 7| = 22$$

4. In the equations above,  $m < 0$  and  $n < 0$ . What is the value of  $m - n$ ?

- (A) 14
- (B) 18
- (C) 20
- (D) 24

$$\frac{1}{4}(12x + 2y) = \frac{27}{4}$$

$$y = 3x$$

5. In the system of equations above, what is the value of  $x$ ?

$$3x + 6y = 14$$

$$2x + 5y = -11$$

6. If  $(x, y)$  is a solution to the system of equations above, what is the value of  $x + y$ ?

- (A) 3
- (B) 17.5
- (C) 24
- (D) 25

$$2n + n + n + n - 6 = 4 + n - 2n$$

7. What is the value of  $n$  in the equation above?

- (A)  $\frac{-3}{4}$

- (B)  $\frac{-2}{3}$
- (C)  $\frac{5}{3}$
- (D)  $\frac{4}{3}$

$$n = 3.65 + 0.35x$$

$$m = 4.25 + 0.15x$$

8. In the equations above,  $m$  and  $n$  represent the heights of bamboo plants, in feet,  $x$  weeks after September 15.

What was the height of the two plants when their heights were equal?

- (A) 1.5
- (B) 3.0
- (C) 4.7
- (D) 4.8

$$4 - 2x \leq 6x - 20$$

9. Which of the following is the solution to the inequality above?

- (A)  $x \leq -3$
- (B)  $x \leq -8$
- (C)  $x \leq 3$
- (D)  $x \geq 3$

10. What value(s) of  $n$  satisfies  $|n + 11| \geq -12$ ?

- (A)  $-1 \leq n \leq 1$
- (B)  $n \geq 11$  or  $n \leq -23$
- (C) There is no value  $n$
- (D) All real numbers

## Answer Explanations

1. **(B)** Simplify to solve for  $x$ .

$$\begin{aligned}
2(-3x - 7) &= 4 - 2x \\
-6x - 14 &= 4 - 2x \\
-6x + 2x - 14 &= 4 - 2x + 2x \\
-4x - 14 &= 4 \\
-4x - 14 + 14 &= 4 + 14 \\
\frac{-4x}{-4} &= \frac{18}{-4} \\
x &= -\frac{18}{4} = -\frac{9}{2}
\end{aligned}$$

2. **(B)** Multiply both sides of the equation by  $\frac{5}{3}$  to isolate  $x$ .

$$\begin{aligned}
\left(\frac{5}{3}\right)\left(\frac{3x}{5}\right) &= (12)\left(\frac{5}{3}\right) \\
x &= 20
\end{aligned}$$

3. **(A)** Substitute 4 for  $y$  and solve for  $x$ .

$$\begin{aligned}
\frac{x+1}{4} &= 4 \\
x+1 &= 16 \\
x &= 15
\end{aligned}$$

4. **(C)** Solve each absolute value equation.

$$|m - 3| = 12$$

Therefore:

$$\begin{aligned}
m - 3 &= 12 \text{ or } m - 3 = -12 \\
m &= 15 \text{ or } m = -9
\end{aligned}$$

Therefore:

$$\begin{aligned}
|n + 7| &= 22 \\
n + 7 &= 22 \text{ or } n + 7 = -22 \\
n &= 15 \text{ or } n = -29
\end{aligned}$$

Since  $m$  and  $n$  are both less than 0, select  $-9$  and  $-29$  for  $m$  and  $n$ , respectively.

$$m - n = -9 - (-29) = 20$$

5.  $\frac{3}{2}$  or 1.5 Multiply both sides of  $\frac{1}{4}(12x + 2y) = \frac{27}{4}$  by 4.

$$\begin{aligned}4\left(\frac{1}{4}(12x + 2y) = \frac{27}{4}\right) \\ 12x + 2y = 27\end{aligned}$$

Substitute  $3x$  for  $y$  and solve for  $x$ .

$$\begin{aligned}12x + (2)(3x) &= 27 \\ 18x &= 27 \\ x &= \frac{3}{2}\end{aligned}$$

6. **(D)** Although the system of equations can be solved by using elimination or substitution, it is easier to subtract the equations.

$$\begin{array}{r}3x + 6y = 14 \\ -(2x + 5y = -11) \\ \hline x + y = 25\end{array}$$

7. **(D)** Combine like terms and isolate the variable.

$$\begin{aligned}2n + n + n + n - 6 &= 4 + n - 2n \\ 5n - 6 &= 4 - n \\ 6n &= 10 \\ n &= \frac{10}{6} = \frac{5}{3}\end{aligned}$$

8. **(C)** Setting  $m = n$  will show the number of weeks when the two plants were the same height.

$$\begin{aligned}n &= 3.65 + 0.35x \\ m &= 4.25 + 0.15x \\ 3.65 + 0.35x &= 4.25 + 0.15x\end{aligned}$$

Solve for  $x$ .

$$\begin{aligned}3.65 + 0.35x &= 4.25 + 0.15x \\0.2x &= 0.6 \\x &= 3\end{aligned}$$

After 3 weeks passed, the two plants were identical heights. Replace  $x$  with 3 in either equation to find the height when they were the same.

$$3.65 + 0.35(3) = 4.7$$

The two plants were both 4.7 feet three weeks after September 15.

9. **(D)** Simplify to solve for  $x$ .

**TIP**

Remember to reverse the direction of the inequality sign when multiplying or dividing an inequality by a negative number.

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$$\begin{aligned}4 - 2x &\leq 6x - 20 \\4 - 2x - 6x &\leq 6x - 20 - 6x \\4 - 8x &\leq -20 \\4 - 4 - 8x &\leq -20 - 4 \\-8x &\leq -24 \\\frac{-8x}{-8} &\leq \frac{-24}{-8} \\x &\geq 3\end{aligned}$$

10. **(D)** The absolute value of a quantity is greater than or equal to 0. Therefore, any value of  $n$  satisfies  $|n + 11| \geq -12$ . The solution set to the inequality, then, is all real numbers.

## Drill 2

For multiple-choice questions, solve the problem and pick the correct answer from the provided choices. Each multiple-choice question has only one correct answer. For student-

produced response questions, solve each problem and write down your answers separately.

$$7x - 23 = 47$$

1. Solve for  $x$  in the equation above.

- (A) 10
- (B) 8
- (C) 6
- (D) 4

2. If  $3(-2x - 6) + 4(4x - 2) = -30$ , what is the value of  $x$ ?

- (A)  $-0.4$
- (B)  $0.8$
- (C)  $2.4$
- (D)  $3.2$

3. If  $\frac{m}{n} = 7$ , what is the value of  $\frac{2n}{m}$ ?

- (A) 14
- (B)  $\frac{7}{2}$
- (C)  $\frac{2}{7}$
- (D)  $\frac{1}{7}$

4. If  $\frac{6}{n} = \frac{18}{n+24}$ , what is the value of  $\frac{n}{12}$ ?

- (A) 1
- (B) 1.5
- (C) 2
- (D) 4.5

5. If the quotient of  $n$  and 8 is 12, what is the value of  $2n - 144$ ?

$$2x - 3y = 24$$

$$x + 4.5y = -12$$

6. In the system of equations above, what is the value of the product of  $x$  and  $y$ ?
- (A) 18
  - (B) 12
  - (C)  $-24$
  - (D)  $-36$
7. Which of the following is not a solution of the inequality  $-7x + 4 \leq 4x - 5$ ?
- (A) 0
  - (B) 1
  - (C) 3
  - (D) 4
8. Ephraim has two summer jobs. The first, as a delivery person, pays \$11.50 per hour. Ephraim's second job, as a math tutor, pays \$15.00 per hour. He wants to work no more than 20 hours per week, and he wants to earn at least \$270.00 weekly. If  $x$  and  $y$  represent the hours he works as a delivery person and as a tutor, respectively, which system of inequalities correctly models the situation?
- (A)  $x + y < 20$   
 $11.50x + 15y > 270$
  - (B)  $x + y \leq 20$   
 $11.50x + 15y \geq 270$
  - (C)  $x + y > 20$   
 $11.50x + 15y \leq 270$
  - (D)  $x + y \geq 270$   
 $11.50x + 15y \leq 20$
9. If  $5x + 3y = 16$  and  $4x + 15y = 26$ , what is the value of  $3x + 6y$ ?
- (A) 15
  - (B) 14
  - (C) 11
  - (D) 7.5

10. If  $f(x) = x^2 + 2$  and  $g(x) = 2x + 3$ , what is the value of  $g(3) - f(-2)$ ?
- (A) 15
  - (B) 9
  - (C) 3
  - (D) 4

### Answer Explanations

1. **(A)** Simplify to solve for  $x$ .

$$\begin{aligned}7x - 23 &= 47 \\7x - 23 + 23 &= 47 + 23 \\7x &= 70 \\x &= 10\end{aligned}$$

2. **(A)** Simplify to find the value of  $x$ .

$$\begin{aligned}3(-2x - 6) + 4(4x - 2) &= -30 \\-6x - 18 + 16x - 8 &= -30 \\10x - 26 &= -30 \\10x &= -4 \\x &= -0.4\end{aligned}$$

3. **(C)** If  $\frac{m}{n} = 7$ , then  $\frac{n}{m} = \frac{1}{7}$ . So  $\frac{2n}{m}$  can be interpreted as  $(2)\left(\frac{n}{m}\right)$ . Since  $\frac{n}{m} = \frac{1}{7}$ , then double that value,  $\frac{2n}{m}$ , must equal  $\frac{2}{7}$ .
4. **(A)** Cross-multiply and solve for  $n$ .

$$\begin{aligned}\frac{6}{n} &= \frac{18}{n+24} \\6(n+24) &= (18)(n) \\6n + 144 &= 18n \\144 &= 12n \\12 &= n\end{aligned}$$

Find the value of  $\frac{n}{12}$  by substituting 12 for  $n$ .

$$\frac{12}{12} = 1$$

5. **48** Solve for  $n$  and then insert into  $2n - 144$ .

$$\frac{n}{8} = 12$$

$$n = 96$$

$$2(96) - 144 = 48$$

6. **(C)** Multiply  $x + 4.5y = -12$  by  $-2$  to attain the same coefficient of  $x$  but with different signs.

Solve by elimination.

$$2x - 3y = 24$$

$$-2(x + 4.5y = -12)$$

Solve by eliminating  $x$ .

$$2x - 3y = 24$$

$$+ -2x - 9y = 24$$

$$\hline -12y = 48$$

$$y = -4$$

Replace  $y$  with  $-4$  in  $2x - 3y = 24$ .

$$2x - 3(-4) = 24$$

$$2x + 12 = 24$$

$$2x = 12$$

$$x = 6$$

The product of  $x$  and  $y$  is  $(6)(-4) = -24$ .

This problem could also have been solved by using the substitution method.

$$x + 4.5y = -12$$

$$x = -4.5y - 12$$

$(-4.5y - 12)$  would be substituted for  $x$  in  $2x - 3y = 24$ .

7. **(A)** Simplify by subtracting  $4x$  from both sides of the inequality.

$$\begin{aligned} -7x + 4 &\leq 4x - 5 \\ -7x + 4 - 4x &\leq 4x - 5 - 4x \\ -11x + 4 &\leq -5 \end{aligned}$$

Subtract 4 from both sides of the inequality.

$$\begin{aligned} -11x + 4 - 4 &\leq -5 - 4 \\ -11x &\leq -9 \end{aligned}$$

Divide both sides of the inequality by  $-11$ , remembering to reverse the direction of the inequality sign.

$$\begin{aligned} \frac{-11x}{-11} &\leq \frac{-9}{-11} \\ x &\geq \frac{9}{11} \end{aligned}$$

8. **(B)** Ephraim will work as many as 20 hours per week but not more. Thus,  $x + y \leq 20$  models the number of hours he wants to work for both jobs. His pay must be at least \$270.00. He'll work  $x$  hours at \$11.50 per hour as a delivery person and  $y$  hours at \$15.00 per hour as a math tutor. Therefore,  $11.50x + 15y \geq 270$  models his weekly earnings. Choice (B) is the only answer that features both of these conditions.
9. **(B)** Although the system of equations can be solved using either the elimination or the substitution method (or even via graphing), a quicker way to calculate the value of  $3x + 6y$  is to add the equations together.

$$\begin{array}{r} 5x + 3y = 16 \\ +4x + 15y = 26 \\ \hline 9x + 18y = 42 \end{array}$$

Next, multiply by  $\frac{1}{3}$ .

$$\begin{aligned}\frac{1}{3}(9x + 18y = 42) \\ 3x + 6y = 14\end{aligned}$$

10. (C) Given that  $g(x) = 2x + 3$ , calculate  $g(3)$ .

$$g(3) = 2(3) + 3 = 9$$

Since  $f(x) = x^2 + 2$ , calculate  $f(-2)$ .

$$f(-2) = (-2)^2 + 2 = 6$$

Thus,  $g(3) - f(-2) = 9 - 6 = 3$ .

Choice (A) is incorrect because it is the sum of  $g(x)$  and  $f(x)$ . Choice (B) is incorrect because it is the value of  $g(3)$ . Choice (D) is incorrect because it is the result of improper calculations.

### Drill 3

For multiple-choice questions, solve the problem and pick the correct answer from the provided choices. Each multiple-choice question has only one correct answer. For student-produced response questions, solve each problem and write down your answers separately.

1. If  $x - 2 = 0.2x$ , what is the value of  $x$ ?
- (A) 2.0
  - (B) 2.5
  - (C) 3.0
  - (D) 3.5

2. If  $4(6 - 2x) - 3(4 + 6x) = -2x + 6$ , what is the value of  $x$ ?

- (A)  $-\frac{1}{4}$
- (B)  $-\frac{1}{8}$
- (C)  $\frac{3}{16}$
- (D)  $\frac{1}{4}$

3. If  $2x$  is 12 less than 48, what is the value of  $2x - 6$ ?

- (A) 16
- (B) 24
- (C) 30
- (D) 36

$$n + 4m = 7$$

4. If  $n = 8$  in the equation above, what is the value of  $m$ ?

- (A)  $-\frac{1}{4}$
- (B)  $-\frac{1}{7}$
- (C) 1
- (D)  $\frac{5}{4}$

5. If  $\frac{7}{2a} = \frac{9}{16}$ , what is the value of  $a$ ?

$$y = 4x - 7$$
$$y = 4x - 2b$$

6. If the system of equations above has infinite solutions, what is the value of  $b$ ?

$$2x + 3y = 4$$
$$5x - y = -7$$

7. At what coordinate pair in the  $xy$ -plane will the two equations above intersect?

- (A) (1, -2)
- (B) (-1, 2)
- (C) (2, 4)
- (D) (4, -4)

$$\begin{aligned}3x + 2y &= 6 \\7x + 4y &= -12\end{aligned}$$

8. Consider the system of equations above. What is the value of  $5x + 3y$ ?
- (A) -3
  - (B) 0
  - (C) 2
  - (D) 3

$$|-2x + 6| < 4$$

9. Which value(s) satisfy the inequality above?
- (A)  $x < 1$  or  $x > 5$
  - (B)  $1 < x < 5$
  - (C)  $-5 < x < -1$
  - (D) There are no values that satisfy this inequality.
10. If  $f(x) = x^2 + 2$  and  $g(x) = 2x - 1$ , what is the value of  $f(g(-2))$ ?

## Answer Explanations

1. **(B)** Simplify to find the value of  $x$ .

$$\begin{aligned}x - 2 &= 0.2x \\x - 2 - x &= 0.2x - x \\-2 &= -0.8x \\\frac{-2}{-0.8} &= \frac{-0.8x}{-0.8} \\2.5 &= x\end{aligned}$$

2. **(D)** Find the value of  $x$  by simplifying.

$$\begin{aligned}
4(6 - 2x) - 3(4 + 6x) &= -2x + 6 \\
24 - 8x - 12 - 18x &= -2x + 6 \\
12 - 26x &= -2x + 6 \\
6 &= 24x \\
\frac{1}{4} &= x
\end{aligned}$$

3. **(C)** The question requires the test taker to translate the words into an algebraic equation. "If  $2x$  is 12 less than 48, what is the value of  $2x - 6$ ?" becomes the following.

$$\begin{aligned}
2x + 12 &= 48 \\
2x &= 36 \\
x &= 18
\end{aligned}$$

Plug in 18 for  $x$  in  $2x - 6$ .

$$2(18) - 6 = 30$$

4. **(A)** Substitute 8 for  $n$  and solve for  $m$ .

$$\begin{aligned}
n + 4m &= 7 \\
8 + 4m &= 7 \\
4m &= -1 \\
m &= -\frac{1}{4}
\end{aligned}$$

5.  $\frac{56}{9}$  or 6.222

Cross-multiply and solve for  $a$ .

$$\begin{aligned}
\frac{7}{2a} &= \frac{9}{16} \\
(7)(16) &= (9)(2a) \\
112 &= 18a \\
6\frac{2}{9} &= a
\end{aligned}$$

This solution will not fit into the digital test form. Transform the solution into an improper fraction or a decimal.

Improper fraction: Multiply 6 by 9 and add 2. Keep 9 as the denominator.

$$\frac{(6 \times 9 + 2)}{9} = \frac{56}{9}$$

Decimal: Divide 56 by 9 = 6.222 . . .

Fill in 6.222 as your final answer.

6. **3.5 or  $\frac{7}{2}$**  If two equations have an infinite number of solutions, their slopes and  $y$ -intercepts are the same. Both equations have the same slope, 4, and must also have the same  $y$ -intercept. Thus, set  $-2b$  equal to  $-7$  and solve for  $b$ .

$$\begin{aligned} -2b &= -7 \\ b &= \frac{7}{2} \end{aligned}$$

The answer can also be expressed as 3.5.

7. **(B)** This system of equations can be solved using either the elimination or the substitution method.

$$\begin{aligned} 2x + 3y &= 4 \\ 5x - y &= -7 \end{aligned}$$

Let's use the elimination method by multiplying the second equation by 3.

$$3(5x - y = -7) \rightarrow 15x - 3y = -21$$

Add the new equation to the top equation.

$$\begin{array}{r}
 2x + 3y = 4 \\
 15x - 3x = -21 \\
 \hline
 17x = -17 \\
 x = -1
 \end{array}$$

You need not solve for  $y$ ; only selection (B) shows  $x = -1$ . You can also use a plug-and-check strategy, although it may be overly time-consuming.

8. **(A)** Notice how the question does not ask for the values of  $x$  and  $y$ . This is a clue that there is a way to bypass the elimination or substitution methods. Add or subtract the equations as presented and see if the result gives a clue to the correct solution. Let's try adding the equations.

$$\begin{array}{r}
 3x + 2y = 6 \\
 7x + 4y = -12 \\
 \hline
 10x + 6y = -6
 \end{array}$$

Divide both sides of  $10x + 6y = -6$  by 2.

$$\begin{array}{r}
 \frac{10x + 6y}{2} = \frac{-6}{2} \\
 5x + 3y = -3
 \end{array}$$

The correct solution to the question is  $-3$ .

9. **(B)** The inequality should be divided into two separate inequalities.

$$\begin{array}{r}
 |-2x + 6| < 4 \\
 -2x + 6 < 4 \qquad -(-2x + 6) < 4 \\
 -2x < -2 \qquad \qquad 2x - 6 < 4 \\
 x > 1 \qquad \qquad \qquad 2x < 10 \\
 \qquad \qquad \qquad \qquad \qquad x < 5
 \end{array}$$

Remember to reverse the direction of the inequality when you divide or multiply by a negative number. The solution is expressed as a

compound inequality.

$$1 < x < 5$$

10. **27** First find  $g(-2)$  by replacing  $x$  with  $-2$  in  $g(x)$ .

$$\begin{aligned}g(x) &= 2x - 1 \\g(-2) &= 2(-2) - 1 = -5\end{aligned}$$

Next, find  $f(-5)$  by replacing  $x$  with  $-5$  in  $f(x)$ .

$$\begin{aligned}f(x) &= x^2 + 2 \\f(-5) &= (-5)^2 + 2 = 27\end{aligned}$$

Thus,  $f(g(-2)) = 27$ .

## Word Problems and Function Interpretation

### Drill 1

For multiple-choice questions, solve the problem and pick the correct answer from the provided choices. Each multiple-choice question has only one correct answer. For student-produced response questions, solve each problem and write down your answers separately.

1. A learning therapist has shown that students can increase reading levels by increasing their required summer reading. She models the increase using the formula  $y = 1.09x + 0.17$ , where  $x$  represents the current grade reading level and  $y$  represents the improvement after the program. Which of the following represents the meaning of 1.09 in the equation?

- (A) The difference between the students who use the program and those who do not
  - (B) The increase in the number of students using the reading program
  - (C) The increase in the reading level plus a constant of 0.17
  - (D) A 17% increase in reading level
2. After researching the future cost of a four-year college education, Sergio and Maria began saving for their daughter's tuition. After two years, they had accrued \$2,870 for their college fund. After five years, the account balance was \$7,150. If Sergio and Maria continue to save at the same rate as they have for the past five years, approximately what will their savings be after 78 months?
- (A) \$7,846.33
  - (B) \$8,413.17
  - (C) \$9,290.02
  - (D) \$9,417.63
3. Ted needs to purchase shirts and belts for the costumes in his school's play. He can buy  $s$  shirts and  $b$  belts, but the quantity may not exceed 100 items. Which of the following inequalities represents the conditions of Ted's purchases?
- (A)  $s + b > 100$
  - (B)  $s + b < 100$
  - (C)  $s + b \leq 100$
  - (D)  $sb \leq 100$
4. Jerry has joined a gymnasium for which he pays dues of \$75 each month plus a daily rate of \$10 for each day that he uses the facilities. The function shown below can be used to determine the cost in dollars per month for being a member of this club, where  $d$  is the number of days.

$$f(d) = 75 + 10d$$

Jerry spent \$155 in January, \$195 in February, and \$225 in March. If he spent a total of \$850 for the months of January, February, March, and April, what was the total number of days that he spent at the club for those four months?

5. A small wooden bridge has a maximum carrying capacity of 15,000 pounds. Three cars are currently crossing the bridge, each with a mean weight of 3,250 pounds. What is the greatest weight, in pounds, a fourth car or truck can have and still cross the bridge safely?
6. Two friends are returning home from Vacaville, CA, to Carlsbad, CA, a round trip of 984 miles. They would like to stop over in Bakersfield to go to the Museum of Art, which has free admission on Fridays. They would also like to eat lunch within walking distance of the museum, spending \$8 each. This side trip would add 54 miles to the trip. They have designated a budget of \$90 for the return trip to Carlsbad. Each gallon of gas costs \$3.64, and their car gets 26 miles per gallon. Do they have enough money to take this extra stop?
  - (A) Yes, they will have \$7.43 extra after the side trip.
  - (B) Yes, they will have \$4.32 extra after the side trip.
  - (C) No, they will need an additional \$2.44.
  - (D) No, they will need an additional \$8.73.
7. Gina runs 28 meters in 4.7 seconds. If she continues to run at this rate, which of the following is the closest to the distance (in meters) she will run in 2.4 minutes?
  - (A) 269 meters
  - (B) 542 meters
  - (C) 653 meters
  - (D) 858 meters
8. An airline company is calculating its passenger cost for a transatlantic flight. The plane flies approximately 3,000 miles at an average air speed of 500 miles per hour. The cost is given by the following function.

$$C(x) = 100 + \frac{x}{10} + \frac{36,000}{x}$$

where  $x$  is the ground speed. Ground speed is defined as air speed  $\pm$  wind speed. What is the airline's cost if there is no wind during the flight?

- (A) \$176
- (B) \$222
- (C) \$358
- (D) \$422

9. A scuba diving instructor charges \$75 per lesson plus an additional fee for the use of her boat. The charge for the use of her boat varies directly with the square root of the time the boat is used. If a lesson plus 25 minutes of boat time costs \$110, what is the total amount charged for a lesson that uses the boat for 36 minutes?

- (A) \$117
- (B) \$122
- (C) \$127
- (D) \$133

10. A food stand sells Mexican fare. Tacos cost \$2.00 each, and enchiladas cost \$4.50 each. On a certain day, revenue from tacos and enchiladas was \$198.00, and the number of tacos and enchiladas sold was 54. How many enchiladas were sold?

- (A) 54
- (B) 36
- (C) 27
- (D) 18

## Answer Explanations

1. **(C)** The learning therapist has demonstrated that her program increases students' reading levels. The coefficient 1.09 in the equation ensures growth. For example, if a student is in the second grade and uses the program, we get the following.

$$x = 2$$

$$y = 1.09(2) + 0.17$$

$$y = 2.35$$

This means a student in second grade would read at a level more than 3 months in advance (assuming a 10-month school year) of students who are not in the reading program.

2. **(C)** The phrase “. . . at the same rate . . .” means use the linear model  $y = mx + b$ . Find the slope by using the ordered pairs (2, \$2,870) and (5, \$7,150).

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7,150 - 2,870}{5 - 2} = \frac{4,280}{3} = 1,426.67$$

Replace  $m$  with 1,426.67.

$$y = 1,426.67x + b$$

Replace  $x$  and  $y$  with either point.

$$2,870 = 2(1,426.67) + b$$

$$16.66 = b$$

$$y = 1,426.67x + 16.66$$

Replace  $x$  with 78 months, but remember to convert it to years first.

$$78 \text{ months} = 6.5 \text{ years}$$

$$y = 1,426.67(6.5) + 16.66$$

$$y = 9,290.02$$

$$\$9,290.02$$

3. **(C)** The number of shirts ( $s$ ) and belts ( $b$ ) cannot exceed 100 items, but they can be equal to that sum. We therefore add the two quantities and ensure that they could not exceed 100 items.

$$s + b \leq 100$$

Choices (A) and (B) can be eliminated at once. The question suggests that the number of items cannot exceed 100 items. That implies the number of items could be equal to 100 but not more. Therefore, the symbol  $\leq$  is needed, but using  $<$  would be incorrect.

4. **55** The dollar amount in April was  $\$850 - \$155 - \$195 - \$225 = \$275$ . The number of days for January can be found by solving the equation  $155 = 75 + 10d$ . Subtract 75 from each side to get  $80 = 10d$ . Then  $d = 8$ . In a similar manner, the number of days for each of February, March, and April can be determined by solving the equations  $195 = 75 + 10d$ ,  $225 = 75 + 10d$ , and  $275 = 75 + 10d$ , respectively. We find that Jerry spent 12 days in February, 15 days in March, and 20 days in April. Thus, the total number of days spent at the club was  $8 + 12 + 15 + 20 = 55$ .
5. **5,250** Find the weight of the three cars currently crossing the bridge. Subtract that sum from 15,000, the carrying capacity of the bridge. Since the average weight of the three cars is 3,250 pounds, multiply that value by 3 to find the weight of all three.

$$3 \times 3,250 = 9,750$$

Subtract 9,750 from 15,000 to find the maximum allowable weight for a fourth car or truck.

$$15,000 - 9,750 = 5,250$$

6. **(C)** The return trip from Vacaville to Carlsbad is 492 miles. Add 54 miles to get a total of 546 miles. Divide this number by 26 miles to the gallon, which comes out to 21 gallons. Multiply 21 by  $\$3.64$ , which comes to  $\$76.44$ . Add  $\$16$  for the two lunches to  $\$76.44$ , which brings the cost to  $\$92.44$ . The friends allocated  $\$90$  for the cost of the return trip, so the side trip and the two lunches will exceed their budget by  $\$2.44$ .

7. **(D)** Use the proportion  $\frac{\text{distance}}{\text{time}} = \frac{\text{distance}}{\text{time}}$  to find how far Gina will run in 2.4 minutes. Since the original ratio is stated in seconds, first convert 2.4 minutes into seconds. Then use the proportion to solve.

$$2.4 \text{ minutes} = 2.4(60) = 144 \text{ seconds}$$

$$\frac{28}{4.7} = \frac{n}{144}$$

$$4.7n = (144)(28)$$

$$4.7n = 4,032$$

$$n \approx 858$$

8. **(B)** Ground speed is defined as air speed  $\pm$  wind speed. If there is no wind, then the ground speed is the air speed because  $500 \pm 0 = 500$ .

$$C(500) = 100 + \frac{500}{10} + \frac{36,000}{500} = 222$$

9. **(A)** The \$75 fee for the lesson is a fixed cost. Subtract that amount from the \$110.00, which is the total charged for the lesson and 25 minutes of time on the boat.

$$\$110 - \$75 = \$35$$

Create a proportion based solely on the time paid for the boat. The question indicates that the fee varies directly with the square root of the time spent on the boat.

$$\frac{\sqrt{\text{time}}}{\text{fee}} = \frac{\sqrt{\text{time}}}{\text{fee}}$$

$$\frac{\sqrt{25}}{35} = \frac{\sqrt{36}}{x}$$

$$\frac{5}{35} = \frac{6}{x}$$

Cross-multiply and solve for  $x$ .

$$5x = 210$$

$$x = 42$$

Add \$42.00, which is the fee for the 36 minutes spent on the boat, to \$75.00, which is the fixed cost of the lesson.

$$\$75 + \$42 = \$117$$

Choices (B), (C), and (D) are incorrect due to erroneous calculations.

10. **(B)** Although a plug-and-check strategy is possible for this problem, using a system of equations is easier.

Let  $x$  = tacos sold

Let  $y$  = enchiladas sold

There were 54 items sold, so the first equation is  $x + y = 54$ . Tacos cost \$2.00, and enchiladas cost \$4.50. The revenue generated was \$198.00, giving us  $2x + 4.5y = 198$ .

Use either elimination or substitution to proceed.

$$\begin{aligned}x + y &= 54 \\y &= 54 - x\end{aligned}$$

Substitute  $54 - x$  for  $y$  in the equation  $2x + 4.5y = 198$ .

$$\begin{aligned}2x + 4.5(54 - x) &= 198 \\2x + 243 - 4.5x &= 198 \\-2.5x &= -45 \\x &= 18\end{aligned}$$

Substitute 18 into  $y = 54 - x$  to find the number of enchiladas sold.

$$y = 54 - 18 = 36$$

## Drill 2

For multiple-choice questions, solve the problem and pick the correct answer from the provided choices. Each multiple-choice question has only one correct answer. For student-produced response questions, solve each problem and write down your answers separately.

1. A marathon runner can run  $m$  miles at  $d$  miles per hour in  $h$  hours. The formula for the distance she runs is  $hd = m$ . If  $m$  is a constant, which of the following conclusions is correct?
  - (A) When  $d$  increases,  $h$  increases.
  - (B) When  $d$  decreases,  $h$  decreases.
  - (C) As  $h$  increases,  $d$  decreases.
  - (D)  $d$  will never increase, but  $m$  can increase or decrease.

2. Tina repairs sinks in a large housing development. Each month, the number of sinks she has to repair is represented by the following equation.

$$B = 360 - 88w$$

where  $B$  is the balance of sinks left to repair and  $w$  is the number of weeks she has worked in a certain month. What is the meaning of 360 in the equation  $B = 360 - 88w$ ?

- (A) Tina will repair 360 sinks each week.
  - (B) Tina starts each month with 360 sinks to repair.
  - (C) Tina repairs 360 sinks a year, almost one per day.
  - (D) Tina repairs 360 sinks daily.
3. Flour costs \$1.50 per pound. What is the cost, in dollars, of  $(2x + 6)$  pounds of flour?

(Ignore the \$ sign.)

- (A)  $\frac{2x + 6}{1.50}$

- (B)  $2x + 7.50$
- (C)  $\frac{1.5}{2x + 6}$
- (D)  $3x + 9$

4. The volume of a certain enclosed gas is shown by the following equation:

$$V = \frac{0.7T}{P} + 3.77$$

where:

$V$  = volume of the gas (in cubic centimeters)

$P$  = pressure (in kilograms per square centimeter)

$T$  = temperature (in Kelvin)

Which of the following expresses temperature in terms of volume and pressure?

- (A)  $T = (P - V)(0.7) - 3.77$
- (B)  $T = \frac{P}{0.7}(V - 3.77)$
- (C)  $T = \frac{P}{(V + 3.77)}$
- (D)  $T = \frac{P}{V - 3.77}$

5. The volume of a certain enclosed gas is shown by the following equation:

$$V = \frac{0.7T}{P} + 3.77$$

where:

$V$  = volume of the gas (in cubic centimeters)

$P$  = pressure (in kilograms per square centimeter)

$T$  = temperature (in Kelvin)

Find the temperature (in Kelvin) when the volume of the gas is 8,000 cubic centimeters and the pressure is 1.2 kg/cm<sup>2</sup>.

- (A) 7,407
- (B) 8,176

- (C) 13,708
- (D) 16,511

6. Belts cost \$11, and blouses cost \$18. If Kara spends at least \$51 but no more than \$62 for  $x$  belts and one blouse, what is one possible value of  $x$ ?
7. Ms. Garcia has a jar containing  $m$  milliliters of a solution for her chemistry class. If she gives 8 milliliters to each student, she will have 6 milliliters left over. If she provides 10 milliliters to each student, she will need an additional 16 milliliters. How many students are in her class?
- (A) 10
  - (B) 11
  - (C) 17
  - (D) 22
8. The formula for the area of a trapezoid is shown by this formula

$$A = \frac{1}{2}(h)(b_1 + b_2)$$

Which of the following expresses  $b_1$  in terms of  $h$ ,  $A$ , and  $b_2$ ?

- (A)  $\frac{2A}{h} - b_2$
- (B)  $\frac{h}{2A} - b_2$
- (C)  $b_2 + 2Ah$
- (D)  $\left(\frac{A}{2h}\right) - b_2$

$n$	1	2	3	4
$r(n)$	-1	1	3	5

9. The table above shows some values of the linear function  $r$ . Which of the following defines  $r$ ?
- (A)  $r(n) = 2n + 3$
  - (B)  $r(n) = 3n - 5$

- (C)  $r(n) = n - 3$   
(D)  $r(n) = 2n - 3$

10. Jamie is considering two different companies to rent him a mobile home trailer for one day. One service charges \$30 per day and \$0.25 per mile. The other service charges \$40 per day and \$0.20 per mile. At what number of miles is the cost for the two services equal?

### Answer Explanations

1. **(A)** Use simple values for  $m$ ,  $d$ , and  $h$  to find the correct conclusion.

Let  $m = 12$

$$hd = m$$

$h$	$d$
1	12
2	6
3	4
4	3
6	2
12	1

As  $h$  increases,  $d$  decreases.

2. **(B)** Tina starts each month with 360 sinks to repair. As each week passes, Tina has 88 fewer sinks to repair within the month.
3. **(D)** Multiply the cost per pound of flour by the number of pounds.

$$(1.50)(2x + 6) = 3x + 9$$

4. **(B)** Isolate  $T$  by subtracting 3.77 from each side of  $V = \frac{0.7T}{P} + 3.77$ .

$$V = \frac{0.7T}{P} + 3.77$$

$$V - 3.77 = \frac{0.7T}{P}$$

Multiply both sides of the equation by  $\frac{P}{0.7}$ , which is the reciprocal of  $\frac{0.7}{P}$ .

$$\left(\frac{P}{0.7}\right)(V - 3.77) = \left(\frac{0.7T}{P}\right)\left(\frac{P}{0.7}\right)$$

$$\left(\frac{P}{0.7}\right)(V - 3.77) = T$$

5. **(C)** Use the formula  $V = \frac{0.7T}{P} + 3.77$ , and input 8,000 for  $V$  and 1.2 for  $P$ .

$$8,000 = \frac{0.7T}{1.2} + 3.77$$

$$7,996.23 = \frac{0.7T}{1.2}$$

Multiply each side of the equation by  $\frac{1.2}{0.7}$ , which is the reciprocal of  $\frac{0.7}{1.2}$ .

$$\left(\frac{1.2}{0.7}\right)(7,996.23) = \left(\frac{0.7T}{1.2}\right)\left(\frac{1.2}{0.7}\right)$$

$$13,708 \approx T$$

6. **3 or 4** Subtract \$18 from \$51 and \$62 to find the amount of money Kara may have spent on belts.

$$51 - 18 = 33$$

$$62 - 18 = 44$$

Kara may have spent between \$33 and \$44 for belts. Given that each belt costs \$11, Kara could have purchased 3 or 4 belts.

7. **(B)** Let  $x$  equal the number of students in the chemistry class. When providing 8 milliliters per student, we get  $m = 8x + 6$ . This reflects the

fact that 6 milliliters are left over from the jar containing  $m$  milliliters. When Ms. Garcia provides 10 milliliters per student, she requires an additional 16 milliliters to ensure each student has the same amount. This situation is represented by the equation  $m = 10x - 16$ . Since  $8x + 6$  and  $10x - 16$  both equal  $m$ , set the expressions equal to one another and solve for  $x$ .

$$10x - 16 = 8x + 6$$

$$2x = 22$$

$$x = 11$$

8. **(A)** Begin to isolate  $b_1$  by multiplying the area formula by 2.

$$A = \frac{1}{2}(h)(b_1 + b_2)$$

$$(2)A = \frac{1}{2}(h)(b_1 + b_2)(2)$$

$$2A = (h)(b_1 + b_2)$$

Divide both sides of the equation by  $h$ , the height of the trapezoid.

$$\frac{2A}{h} = \frac{(h)(b_1 + b_2)}{h}$$

$$\frac{2A}{h} = b_1 + b_2$$

Subtract  $b_2$  from both sides of the equation.

$$\frac{2A}{h} - b_2 = b_1$$

9. **(D)** Use the formula  $\frac{\text{rise}}{\text{run}}$  to find the slope of the linear function. As  $n$  increases by 1,  $r(n)$  increases by 2. Thus,  $r(n) = 2n + b$ , with  $b$  representing the  $y$ -intercept. Input any of the points to calculate  $b$ .

$$(4, 5)$$

$$5 = 2(4) + b$$

$$5 = 8 + b$$

$$-3 = b$$

$$r(n) = 2n - 3$$

10. **200** Let  $x$  = the number of miles for the two trailer rental costs to be equal.

$$40 + 0.20x = 30 + 0.25x$$

$$40 + 0.20x - 0.20x = 30 + 0.25x - 0.20x$$

$$40 = 30 + 0.05x$$

$$10 = 0.05x$$

$$200 = x$$

### Drill 3

For multiple-choice questions, solve the problem and pick the correct answer from the provided choices. Each multiple-choice question has only one correct answer. For student-produced response questions, solve each problem and write down your answers separately.

1. The term *light-year* refers to the distance light travels in one year. If light travels 186,000 miles per second, which of the following is closest to the distance from Earth to Sirius, 8 light-years distant?
  - (A) 47 billion miles
  - (B) 198 billion miles
  - (C) 9 trillion miles
  - (D) 50 trillion miles
2. The term *light-year* refers to the distance light travels in one year. If light travels 186,000 miles per second, how far does light travel in

one-billionth of a second? (Note: 1 mile = 5,280 feet)

- (A) 10 miles
- (B) 1 mile
- (C) 1 foot
- (D) 0.01 feet

$x$	$g(x)$
0	6
-2	2
-3	0
-4	-2

3. The function  $g$  is defined by a certain polynomial. Some of the values of  $g(x)$  are shown in the table above. Which of the following must be a factor of  $g(x)$ ?

- (A)  $x - 6$
- (B)  $x - 3$
- (C)  $x + 3$
- (D)  $x + 6$

4. The density of an object is found by dividing the object's mass by its volume. If the density of an object is 4 grams per milliliter and its mass is 30 grams, what is the volume in milliliters of the object?

- (A) 4.8
- (B) 6.1
- (C) 7.5
- (D) 9.2

$x$	$f(x)$
0	5
2	3
4	1
5	0

8	-2
---	----

5. The function  $f$  is defined by a polynomial. Some values of  $x$  and  $f(x)$  are shown in the table above. Which of the following must be a factor of  $f(x)$ ?
- (A)  $x$
  - (B)  $x - 3$
  - (C)  $x - 5$
  - (D)  $x - 8$

$$F = \frac{9}{5}C + 32$$

6. The equation above shows how a temperature,  $C$ , measured in degrees Celsius, relates to a temperature,  $F$ , measured in degrees Fahrenheit. Based on the equation, which of the following must be true?
- I. An increase in temperature of 1 degree Celsius is equivalent to an increase of 1.8 degrees Fahrenheit.
  - II. A temperature of  $5^\circ\text{F}$  is equivalent to  $-15^\circ\text{C}$ .
  - III. A temperature of  $20^\circ\text{C}$  is equivalent to  $62^\circ\text{F}$ .
- (A) I only
  - (B) I and II
  - (C) I and III
  - (D) III only

7. A manufacturer of surgical equipment measures its profit by using the following function, where  $x$  is the number of units the company sells in a three-month period.

$$P(x) = 11,000x - 6,750$$

If the company's profit over a three-month period was \$9,321,250, how many units did it sell?

8. The golden ratio compares dimensions that are appealing to the eye. Sometimes the golden ratio appears in nature but more frequently appears in painting and architecture. The ratio has been found to be  $\frac{1+\sqrt{5}}{2}:1$ , which corresponds to length:width.

A new downtown cultural center is being planned for construction in 2027. The city planners hope to construct the main amphitheater using the golden ratio. If the proposed width of the amphitheater is to be 87 meters, what will be its length? (Round your answer to the nearest meter.)

- (A) 211  
(B) 176  
(C) 157  
(D) 141
9. In a homecoming game, a girls' basketball team scored  $\frac{1}{3}$  of its points during the first quarter,  $\frac{1}{4}$  of its points during the second quarter,  $\frac{1}{3}$  of its points in the third quarter, and 8 points in the fourth quarter. How many points did the team score in the game?
- (A) 84  
(B) 90  
(C) 96  
(D) 102
10. Conner needs to plan a production budget for a computer firm. In order to maximize profit, he needs to buy  $m$  computer frames at \$18.75 and  $r$  keyboards at \$13.25 each. If the cost of this transaction must be less than \$139.00, which of the following inequalities represents Conner's model for profit maximization?
- (A)  $18.75m + 13.25r < 139$   
(B)  $18.75m + 13.25r \leq 139$   
(C)  $18.75r + 13.25m < 139$   
(D)  $18.75r + 13.25m \leq 139$

## Answer Explanations

1. **(D)** Find the number of seconds in one year.

$$(365 \text{ days}) \times (24 \text{ hours in a day}) \times (60 \text{ minutes in an hour}) \times (60 \text{ seconds in a minute}) = 31,536,000 \text{ seconds in one year}$$

Next, find the product of 31,536,000 and 186,000 to find how many miles light travels in one year.

$$186,000 \times 31,536,000 = 5,860,000,000,000 \text{ (about 6 trillion miles)}$$

Multiply 5,860,000,000,000 by 8 to find the number of miles to Sirius.

$$5,860,000,000,000 \times 8 = 46.9 \text{ trillion}$$

Choice (D), 50 trillion miles, is the closest to this figure.

2. **(C)** Divide 186,000 miles by 1 billion. Scientific notation will facilitate this process.

$$\frac{1.86 \times 10^5}{1 \times 10^9} = 1.86 \times 10^{-4} = 0.000186$$

In one-billionth of a second, light travels 0.000186 miles. Multiply 0.000186 by 5,280 feet, which is the number of feet in a mile.

$$0.000186 \times 5,280 = 0.98 \text{ foot}$$

The distance 0.98 foot is closest to choice (C), 1 foot.

3. **(C)** Note that when  $x = -3$ ,  $y = 0$ . When a function intersects the  $x$ -axis in the  $xy$ -plane, then  $y = 0$ ; that  $x$ -value is a root of the function. We therefore conclude that  $x = -3$  is a root of the function. Adding 3 to both sides yields  $x + 3 = 0$ .

4. **(C)** The formula for the density of an object is presented as  $\text{density} = \frac{\text{mass}}{\text{volume}}$ . Input 4 grams per milliliter for the density and 30 grams for mass to find the volume.

$$4 = \frac{30}{V}$$

Multiply both sides of the equation by  $V$ .

$$\begin{aligned}(V)(4) &= \left(\frac{30}{V}\right)(V) \\ 4V &= 30 \\ V &= 7.5\end{aligned}$$

5. **(C)** A polynomial has a zero  $x$  if  $f(x) = 0$ . The chart indicates  $(5, 0)$  satisfies the function, so  $f(5) = 0$  and  $x = 5$ . By subtracting 5 from both sides of the equation, we get  $x - 5 = 0$ . Therefore,  $x - 5$  is a factor of  $f(x)$ .
6. **(B)** Verify the accuracy of the statements.

I. An increase in temperature of 1 degree Celsius is equivalent to an increase of 1.8 degrees Fahrenheit.

Use  $C = 1$  and  $C = 2$

$$\begin{aligned}\frac{9}{5}(1) + 32 &= 33.8 \\ \frac{9}{5}(2) + 32 &= 35.6 \\ 35.6 - 33.8 &= 1.8\end{aligned}$$

Statement I is correct.

II. A temperature of  $5^{\circ}\text{F}$  is equivalent to  $-15^{\circ}\text{C}$ .

$$\begin{aligned}5 &= \frac{9}{5}C + 32 \\ -27 &= \frac{9}{5}C \\ -15 &= C\end{aligned}$$

Statement II is correct.

III. A temperature of  $20^{\circ}\text{C}$  is equivalent to  $62^{\circ}\text{F}$ .

$$\frac{9}{5}(20) + 32 = 68$$

Statement III is incorrect. Statements I and II are correct and statement III is incorrect.

7. **848** Replace  $P(x)$  with  $\$9,321,250$ , which is the company's profit.

$$9,321,250 = 11,000x - 6,750$$

$$9,328,000 = 11,000x$$

$$848 = x$$

8. **(D)** Use a proportion to find the length of the proposed amphitheater in the downtown cultural center. This operation can be facilitated by approximating the length in decimal form:  $\frac{1 + \sqrt{5}}{2}$  is approximately equal to 1.62:1. Use this ratio, substituting 87 for the width.

$$\frac{1.62}{1} = \frac{x}{87}$$

Cross-multiply and solve for  $x$ .

$$x = (1.62)(87) = 140.94$$

When 140.94 is rounded to the nearest meter, the result is 141.

Choice (A) can be eliminated quickly (and possibly choice (B) as well). Once it is determined that the golden ratio is about 1.62:1, 211 would surely seem more than twice as large as 87.

9. **(C)** Let  $x$  = the number of points scored in the game.

$$\begin{aligned}\frac{1}{3}x + \frac{1}{4}x + \frac{1}{3}x + 8 &= x \\ \frac{11}{12}x + 8 &= x \\ 8 &= \frac{1}{12}x \\ 96 &= x\end{aligned}$$

10. **(A)** The cost for  $m$  computers at \$18.75 each is \$18.75 $m$ . The cost for  $r$  keyboards at \$13.25 each is \$13.25 $r$ . The cost must be less than \$139, so we arrive at the following inequality.

$$18.75m + 13.25r < 139$$

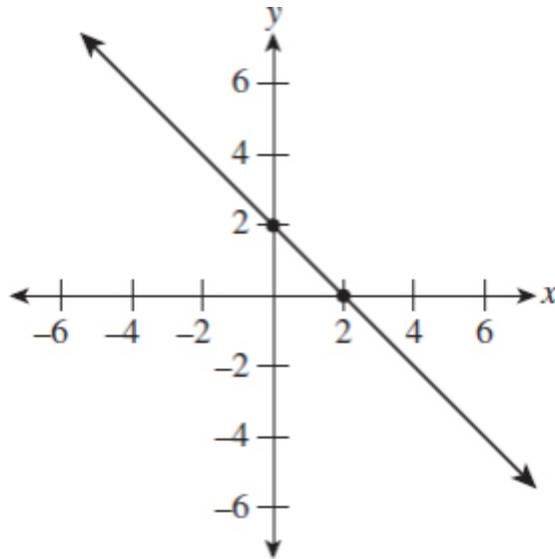
Choices (B) and (D) can automatically be eliminated because each features  $\leq$  (less than or equal to). The problem stipulates the cost must be less than \$139.

## Lines and Slopes

### Drill 1

For multiple-choice questions, solve the problem and pick the correct answer from the provided choices. Each multiple-choice question has only one correct answer. For student-produced response questions, solve each problem and write down your answers separately.

1. Which of the following is an equation for the line graphed above?



- (A)  $y = -x + 2$
- (B)  $y = x + 2$
- (C)  $y = 2$
- (D)  $x - y = 2$

$$y = -\frac{5}{8}x + 3$$

2. Which of the following is an equation of a line that is parallel to the equation above?

- (A)  $5x + 8y = 48$
- (B)  $5x - 8y = 24$
- (C)  $6x + 3y = 22$
- (D)  $3x - 5y = 17$

3. Line A is perpendicular to a line that is parallel to  $y = \frac{5}{9}x + 7$ . What is the slope of line A?

- (A)  $\frac{5}{9}$
- (B)  $-\frac{5}{9}$
- (C)  $\frac{9}{5}$
- (D)  $-\frac{9}{5}$

4. Which of the following equations represents a line that is parallel to the line with the equation  $y = \frac{3}{2}x - \frac{7}{4}$ ?
- (A)  $6x + 4y = 8$   
(B)  $-6x + 4y = 11$   
(C)  $x - 2y = -4$   
(D)  $3x + 2y = 7$
5. A line passes through the points  $(x, 6)$  and  $(9, 0)$ . If the slope of the line connecting the two points is  $\frac{2}{3}$ , what is the value of  $x$ ?
6. In a linear function,  $f(4) = 6$  and  $f(-2) = 14$ . What is the value of  $f(9)$ ?
- (A)  $\frac{20}{3}$   
(B)  $\frac{2}{3}$   
(C)  $-\frac{2}{3}$   
(D)  $-3$
7. In the  $xy$ -plane, the line determined by  $(6, m)$  and  $(m, 54)$  passes through the origin. Which of the following could be the value of  $m$ ?
- (A) 2  
(B) 18  
(C) 36  
(D) 48

$x$	$f(x)$
-2	-2.2
0	-3
6	-5.4
7	-5.8

8. Some values of the linear function  $f$  are shown in the table. What is the value of  $f(8)$ ?

- (A)  $-0.6$   
(B)  $-1.8$   
(C)  $-3.2$   
(D)  $-6.2$
9. Given the equation in the  $xy$ -plane  $-5x + 6y = -7$ , which of the following equations is perpendicular to the graph of the equation above?
- (A)  $-6x + 10y = 7$   
(B)  $6x + 5y = 15$   
(C)  $6x - 20y = 15$   
(D)  $5x + 6y = -18$
10. Line  $\overleftrightarrow{AB}$  has a slope of  $\frac{7}{2}$ . If the line passes through  $(-6, 3)$  and  $(x, -3)$ , what is the value of  $x$ ?
- (A)  $\frac{2}{7}$   
(B)  $-\frac{37}{57}$   
(C)  $-\frac{8}{7}$   
(D)  $-\frac{54}{7}$

## Answer Explanations

1. **(A)** The line crosses the  $y$ -axis at  $(0, 2)$ . So  $b$ , which is the  $y$ -intercept, is 2. Using  $\frac{\text{rise}}{\text{run}}$  to identify the slope, we go down 1 unit and move to the right 1 unit. Thus, the slope,  $m$ , equals  $-1$ . With the slope equaling  $-1$  and the  $y$ -intercept at 2, the equation of the line in the graph is  $y = -x + 2$ .
2. **(A)** Parallel lines have the same slope but different  $y$ -intercepts. The slope of  $y = -\frac{5}{8}x + 3$  is  $-\frac{5}{8}$ , and the  $y$ -intercept is 3. Transform each of the answer choices into slope-intercept form to assess slopes and  $y$ -intercepts. Choice (A) is  $5x + 8y = 48$ . Begin the transformation by isolating  $y$ .

$$\begin{aligned}
 5x + 8y &= 48 \\
 8y &= -5x + 48 \\
 y &= -\frac{5}{8}x + 6
 \end{aligned}$$

Rewriting  $5x + 8y = 48$  in slope-intercept form yields the same slope,  $-\frac{5}{8}$ , but a different  $y$ -intercept, 6. So, choice (A) is parallel to the given equation.

3. **(D)** Parallel lines have the same slope. Since the slope of  $y = \frac{5}{9}x + 7$  is  $\frac{5}{9}$ , then a line parallel to it will also have a slope that measures  $\frac{5}{9}$ . Perpendicular lines have slopes that are the opposite reciprocals of one another. Thus, a line perpendicular to a line with slope of  $\frac{5}{9}$  will have a slope of  $-\frac{9}{5}$ .
4. **(B)** Parallel lines have the same slope but distinct  $y$ -intercepts. The equation  $y = \frac{3}{2}x - \frac{7}{4}$  has slope  $\frac{3}{2}$  and  $y$ -intercept  $-\frac{7}{4}$ . Transform each of the equations to slope-intercept form,  $y = mx + b$ , to compare slopes and  $y$ -intercepts. Choice (B) offers the following:

$$-6x + 4y = 11$$

Transform to slope-intercept form by isolating  $y$ .

$$\begin{aligned}
 -6x + 6x + 4y &= 6x + 11 \\
 4y &= 6x + 11
 \end{aligned}$$

Divide both sides of the equation by 4.

$$\begin{aligned}
 \frac{4}{4}y &= \frac{6}{4}x + \frac{11}{4} \\
 y &= \frac{3}{2}x + \frac{11}{4} \\
 m &= \frac{3}{2} \text{ and } b = \frac{11}{4}
 \end{aligned}$$

The equations  $y = \frac{3}{2}x + \frac{11}{4}$  and  $y = \frac{3}{2}x - \frac{7}{4}$  have the same slope but different  $y$ -intercepts, so the two lines are parallel.

5. **18** The slope of a line can be found by using the formula  $\frac{\text{rise}}{\text{run}}$  or  $\frac{y_2 - y_1}{x_2 - x_1}$ . Substitute (9, 0) for  $x_2$  and  $y_2$  and (x, 6) for  $x_1$  and  $y_1$ . Set the value of that fraction to  $\frac{2}{3}$ , which is the slope of the line connecting these two points.

$$\frac{0 - 6}{9 - x} = \frac{2}{3}$$
$$\frac{-6}{9 - x} = \frac{2}{3}$$

Cross-multiply and solve for x.

$$2(9 - x) = (-3)(6)$$
$$18 - 2x = -18$$
$$-2x = -36$$
$$x = 18$$

6. **(C)** A linear function that contains  $f(4) = 6$  and  $f(-2) = 14$  indicates that the points (4, 6) and (-2, 14) satisfy the function. Use these points to find the slope of the function.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{14 - 6}{-2 - 4} = \frac{-4}{3}$$

Use the point-slope form of a line to create the linear function.

$$y - y_1 = m(x - x_1)$$

Substitute either point for x and y.

$$y - 6 = \frac{-4}{3}(x - 4)$$

Substitute 9 for x to find  $f(9)$ .

$$y - 6 = \frac{-4}{3}(9 - 4)$$

$$y - 6 = -\frac{20}{3}$$

$$y = \frac{-2}{3}$$

Therefore,  $f(9) = \frac{-2}{3}$ .

7. **(B)** In order for the line containing  $(6, m)$  and  $(m, 54)$  to pass through the origin, each point must share the slope that connects it to the origin. Use the formula  $\frac{\text{rise}}{\text{run}}$  for both points, and set the quantities equal.

$$\frac{m}{6} = \frac{54}{m}$$

Cross-multiply and solve for  $m$ .

$$m^2 = 324$$

$$m = 18 \text{ or } m = -18$$

Although both 18 and  $-18$  are solutions to the equation, only 18 was provided as an answer choice.

8. **(D)** Find the slope of  $f(x)$  by selecting values from the table, such as  $(-2, -2.2)$  and  $(0, -3)$ .

$$m = \frac{-3 - (-2.2)}{0 - (-2)} = -0.4$$

The table shows the coordinates  $(0, -3)$ , which means that the  $y$ -intercept is  $-3$ . Thus, the linear function  $f$  is defined as  $f(x) = -0.4x - 3$ .

Substitute 8 for  $x$ .

$$f(8) = -0.4(8) - 3$$

$$f(8) = -6.2$$

9. **(B)** The slopes of perpendicular lines are the opposite reciprocal of one another. Transform  $-5x + 6y = -7$  to slope-intercept form to determine its slope.

$$\begin{aligned} -5x + 6y &= -7 \\ 6y &= 5x - 7 \\ y &= \frac{5}{6}x - \frac{7}{6} \end{aligned}$$

The slope of the line  $-5x + 6y = -7$  is  $\frac{5}{6}$ . The slope of a line perpendicular to  $-5x + 6y = -7$  is  $-\frac{6}{5}$ . Only choice (B) has slope  $-\frac{6}{5}$ .

$$\begin{aligned} 6x + 5y &= 15 \\ 5y &= -6x + 15 \\ y &= -\frac{6}{5}x + 3 \end{aligned}$$

When expressed in slope-intercept form, the slope of  $6x + 5y = 15$  is  $-\frac{6}{5}$ . Choices (A), (C), and (D) are incorrect because their slopes are  $\frac{3}{5}$ ,  $\frac{3}{10}$ , and  $-\frac{6}{5}$ , respectively.

10. **(D)** Use the slope formula to solve for  $x$ .

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ \frac{-3 - 3}{x - (-6)} &= \frac{7}{2} \\ \frac{-6}{x + 6} &= \frac{7}{2} \end{aligned}$$

Cross-multiply and solve for  $x$ .

$$\begin{aligned} (2)(-6) &= 7(x + 6) \\ -12 &= 7x + 42 \\ -54 &= 7x \\ -\frac{54}{7} &= x \end{aligned}$$

## Drill 2

For multiple-choice questions, solve the problem and pick the correct answer from the provided choices. Each multiple-choice question has only one correct answer. For student-produced response questions, solve each problem and write down your answers separately.

1. What is the slope of a line that passes through  $(6, -4)$  and  $(-12, 2)$ ?
  - (A)  $-\frac{1}{3}$
  - (B)  $-\frac{1}{6}$
  - (C)  $\frac{1}{2}$
  - (D)  $\frac{2}{3}$
2. What is the slope-intercept form of a line passing through  $(6, -4)$  and  $(-12, 2)$ ?
  - (A)  $y = -\frac{1}{3}x - 6$
  - (B)  $y = -\frac{1}{3}x - 4$
  - (C)  $y = -\frac{1}{3}x - 2$
  - (D)  $y = -\frac{1}{3}x$
3. The line  $6x + 4y = -3$  is written in standard form. What is the slope of a line that is parallel to  $6x + 4y = -3$ ?
  - (A)  $\frac{3}{2}$
  - (B)  $\frac{3}{4}$
  - (C)  $-\frac{2}{3}$
  - (D)  $-\frac{3}{2}$
4. What is the slope of a line that is perpendicular to  $y = -\frac{3}{2}x - \frac{3}{4}$ ?

- (A)  $-\frac{2}{3}$
- (B)  $-\frac{2}{3}$
- (C)  $\frac{2}{3}$
- (D)  $\frac{4}{3}$

5. The lines  $y = \frac{5}{8}x - 7$  and  $4x - 3y = 7$  intersect at how many points?
- (A) 0
  - (B) 1
  - (C) Infinitely many points
  - (D) There is not enough information to make a conclusion.
6. The lines  $4x - 7y = 5$  and  $7x + 4y = 3$  intersect at how many points?
- (A) 0
  - (B) 1
  - (C) Infinitely many points
  - (D) There is not enough information to make a conclusion.
7. The lines  $y = \frac{5}{3}x - 3$  and  $y = \frac{5}{3}x - 5$  intersect at how many points?
- (A) 0
  - (B) 1
  - (C) Infinitely many points
  - (D) There is not enough information to make a conclusion.
8. The lines  $3x + 2y = 6$  and  $9x + 6y = 18$  intersect at how many points?
- (A) 0
  - (B) 1
  - (C) Infinitely many points
  - (D) There is not enough information to make a conclusion.
9. The slope of  $\overleftrightarrow{RS}$  is  $-\frac{7}{5}$ . If the line passes through  $(6, -10)$  and  $(4, y)$ , what is the value of  $y$ ?
10. What is the  $x$ -intercept of the line  $y = -\frac{2}{3}x - 6$ ?

- (A)  $(-9, 0)$
- (B)  $(0, -6)$
- (C)  $(4, 0)$
- (D)  $(8, 0)$

### Answer Explanations

1. **(A)** Calculate the slope,  $m$ , by using the slope formula and the points  $(6, -4)$  and  $(-12, 2)$ .

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$\frac{2 - (-4)}{-12 - 6} = \frac{6}{-18} = -\frac{1}{3}$$

2. **(C)** Find the slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$\frac{2 - (-4)}{-12 - 6} = \frac{6}{-18} = -\frac{1}{3}$$

Use this information to find  $b$ , the  $y$ -intercept.

$$y = -\frac{1}{3}x + b$$

Use  $(6, -4)$  or  $(12, -2)$  to calculate  $b$ . Let's use  $(-12, 2)$ . Substitute  $-12$  for  $x$  and  $2$  for  $y$ .

$$2 = \left(-\frac{1}{3}\right)(-12) + b$$
$$2 = 4 + b$$
$$-2 = b$$

In slope-intercept form, the equation of a line that passes through  $(6, -4)$  and  $(12, -2)$  is the following.

$$y = -\frac{1}{3}x - 2$$

3. **(D)** Parallel lines have the same slope. Convert  $6x + 4y = -3$  into slope-intercept form to find the slope.

$$\begin{aligned}6x + 4y &= -3 \\4y &= -6x - 3 \\y &= -\frac{6}{4}x - \frac{3}{4} \\y &= -\frac{3}{2}x - \frac{3}{4}\end{aligned}$$

The slope of  $6x + 4y = -3$  is  $-\frac{3}{2}$ . A parallel line would also have a slope of  $-\frac{3}{2}$ .

4. **(C)** Perpendicular lines have slopes that are the negative (or opposite) reciprocals. The line  $y = -\frac{3}{2}x - \frac{3}{4}$  has a slope that equals  $-\frac{3}{2}$ . The negative reciprocal of  $-\frac{3}{2}$  is  $\frac{2}{3}$ .
5. **(B)** Find the slope of each line.

$$\begin{aligned}y &= \frac{5}{8}x - 7 \\m &= \frac{5}{8} \\4x - 3y &= 7 \\-3y &= -4x + 7 \\y &= \frac{4}{3}x - \frac{7}{3} \\m &= \frac{4}{3}\end{aligned}$$

Lines that have different slopes intersect at 1 point.

6. **(B)** Transform each line from standard form to slope-intercept form to compare the slopes.

$$\begin{aligned}
 4x - 7y &= 5 \\
 -7y &= -4x + 5 \\
 y &= \frac{4}{7}x - \frac{5}{7}
 \end{aligned}$$

$$\begin{aligned}
 7x + 4y &= 3 \\
 4y &= -7x + 3 \\
 y &= -\frac{7}{4}x + \frac{3}{4}
 \end{aligned}$$

The slopes of the lines,  $\frac{4}{7}$  and  $-\frac{7}{4}$ , are negative reciprocals of each other, so the lines are perpendicular. Perpendicular lines intersect at 1 point.

7. **(A)** The lines have the same slope but different  $y$ -intercepts. Lines with the same slope and different  $y$ -intercepts are parallel and thus do not intersect.
8. **(C)**  $3x + 2y = 6$  and  $9x + 6y = 18$  are the same line.

$$\begin{aligned}
 (3)(3x + 2y &= 6) \\
 9x + 6y &= 18
 \end{aligned}$$

Since the two equations represent the same line, they intersect at an infinite number of points.

9.  $-\frac{36}{5}$  or  $-7.2$

Use the slope formula to calculate the value of  $y$ .

$$\begin{aligned}
 m &= \frac{y_2 - y_1}{x_2 - x_1} \\
 -\frac{7}{5} &= \frac{y - (-10)}{4 - 6} \\
 -\frac{7}{5} &= \frac{y + 10}{-2}
 \end{aligned}$$

Cross-multiply and solve for  $y$ .

$$\begin{aligned}(-7)(-2) &= 5(y + 10) \\ 14 &= 5y + 50 \\ -36 &= 5y \\ -\frac{36}{5} &= y\end{aligned}$$

The fraction  $-\frac{36}{5}$  can be expressed as  $-7.2$ .

10. **(A)** You can use a plug-and-check strategy to solve this problem quickly. Plug in  $(9, 0)$  for  $x$  and  $y$  in the equation  $y = -\frac{2}{3}x - 6$ .

$$\begin{aligned}0 &= \left(-\frac{2}{3}\right)(-9) - 6 \\ 0 &= 0\end{aligned}$$

Selection (B),  $(0, -6)$ , is the  $y$ -intercept.

### TIP

Although some cubic equations may occur on the SAT, the majority of nonlinear functions are quadratic. When graphed, a quadratic equation will be a parabola in the  $xy$ -plane. It's important to know how to graph the  $x$ - and  $y$ -intercepts and the vertex of a parabola. In addition, the test taker must know how to translate a parabola up or down and left or right. These practice questions will help you master those translations.

## Advanced Math

The Advanced Math portion of the digital SAT Math section will require the test taker to be proficient in the following areas:

- Equivalent expressions
- Nonlinear equations in one variable
- Systems of equations in two variables
- Nonlinear functions

## Polynomials and Factoring

## Drill 1

For multiple-choice questions, solve the problem and pick the correct answer from the provided choices. Each multiple-choice question has only one correct answer. For student-produced response questions, solve each problem and write down your answers separately.

1. If  $m = 2x^3 + 3x - 11$  and  $n = 4x^2 + 5x - 7$ , what is  $3n - m$  in terms of  $x$ ?
- (A)  $-2x^3 - 4x^2 - 2x - 18$   
(B)  $-(2x^3 + 3x^2 - 5x - 6)$   
(C)  $2x^3 - 12x^2 - 12x + 10$   
(D)  $-2x^3 + 12x^2 + 12x - 10$

$$2(2x^2 - 7.5x + 3) - (-4x^2 + 2x - 7)$$

2. If the expression above is written in the form of  $ax^2 + bx + c$ , what is the value of  $a + b + c$ ?

$$3x^2 + 3x - 36 = 0$$

3. What are all of the values that satisfy the quadratic equation above?
- (A)  $x = -3$  and  $x = 4$   
(B)  $x = -4$  and  $x = 3$   
(C)  $x = 3$  and  $x = 4$   
(D)  $x = -3$  and  $x = -4$

$$-4xy^2 - 2xy + xy - 8mn + mn$$

4. Which of the following is equivalent to the expression above?
- (A)  $-5x^3y^3 - 8m^2n^2$

- (B)  $-5x^3y - 7m^2n^2$
- (C)  $32y - 8$
- (D)  $-4xy^2 - xy - 7mn$

5. Which of the following is equivalent to  $\frac{1}{\frac{1}{x+4} + \frac{1}{x+6}}$ ?

- (A)  $x^2 + 10x + 24$
- (B)  $\frac{1}{x+12}$
- (C)  $\frac{x^2 + 10x + 24}{2x + 10}$
- (D)  $\frac{x+4}{x+6}$

$$f(x) = \frac{1}{2(x-3)^2 + 4(x-3) + 2}$$

6. For what value of  $x$  is the function  $f$  undefined in the equation above?

7. If  $m^2 - 6m + 8 = 0$ , what is one possible value of  $\frac{1}{m}$ ?

$$(-4x^3y - xy^3 + 4x) - (7x - 4xy^3 + 2x^2)$$

8. Which of the following is equivalent to the expression above?

- (A)  $-4x^3y + 3xy^3 - 2x^2 - 3x$
- (B)  $6x^3y^3 - 3x$
- (C)  $-4x^3y + 3xy^3 + 2x^2 - 11x$
- (D)  $-9x^3y^3 + 3x$

9. If  $x^2 + y^2 = 12$  and  $xy = -13$ , what is the value of  $2(x - y)^2$ ?

- (A)  $-26$
- (B)  $1$
- (C)  $38$
- (D)  $76$

10. If  $\frac{m^{x^2}}{m^{y^2}} = m^{36}$ ,  $m > 1$ , and  $x + y = 9$ , what is the value of  $x - y$ ?

- (A) 4
- (B) 6
- (C) 9
- (D) 16

## Answer Explanations

1. **(D)** Substituting for  $m$  and  $n$  into  $3n - m$  results in the following:

$$\begin{aligned} 3(4x^2 + 5x - 7) - (2x^3 + 3x - 11) &= \\ 12x^2 + 15x - 21 - 2x^3 - 3x + 11 & \end{aligned}$$

Combine like terms by adding and subtracting those terms with variables raised to the same power.

$$-2x^3 + 12x^2 + 12x - 10$$

2. **4** First multiply  $2(2x^2 - 7.5x + 3)$ . Next multiply each term in the second polynomial by  $-1$ . Then combine like terms.

$$\begin{aligned} 2(2x^2 - 7.5x + 3) - (-4x^2 + 2x - 7) & \\ 4x^2 - 15x + 6 + 4x^2 - 2x + 7 & \\ 8x^2 - 17x + 13 & \end{aligned}$$

The expression  $8x^2 - 17x + 13$  yields  $a = 8$ ,  $b = -17$ , and  $c = 13$ .

$$a + b + c = 8 - 17 + 13 = 4$$

3. **(B)** Simplify the equation and factor.

$$\begin{aligned} \frac{1}{3}(3x^2 + 3x - 36) &= 0 \\ x^2 + x - 12 &= 0 \\ (x + 4)(x - 3) &= 0 \end{aligned}$$

Set each factor equal to 0 and solve for  $x$ .

$$\begin{array}{l} x + 4 = 0 \quad x - 3 = 0 \\ x = -4 \quad x = 3 \end{array}$$

4. **(D)** Combine like terms by adding and subtracting the coefficients. Like terms are those that contain the same variable(s) raised to the same power(s). In this example, the following are the like terms.

$$\begin{array}{l} -2xy, xy \text{ and } -8mn, mn \\ -4xy^2 - 2xy + xy - 8mn + mn = -4xy^2 - xy - 7mn \end{array}$$

Remember, a term such as  $mn$  is understood to have a coefficient of 1.

5. **(C)** Multiply  $\frac{1}{x+4}$  by  $\frac{x+6}{x+6}$  and  $\frac{1}{x+6}$  by  $\frac{x+4}{x+4}$ .

$$\begin{aligned} & \frac{1}{x+4} \times \frac{(x+6)}{(x+6)} + \frac{1}{x+6} \times \frac{(x+4)}{(x+4)} = \\ & \frac{1}{x+6+x+4} = \\ & \frac{1}{(x+4)(x+6)} = \\ & \frac{1}{2x+10} = \\ & \frac{x^2+10x+24}{2x+10} \end{aligned}$$

6. **2** Simplify the denominator.

$$\begin{aligned} f(x) &= \frac{1}{2(x-3)^2 + 4(x-3) + 2} \\ f(x) &= \frac{1}{2x^2 - 12x + 18 + 4x - 12 + 2} \\ f(x) &= \frac{1}{2x^2 - 8x + 8} \\ f(x) &= \frac{1}{2(x^2 - 4x + 4)} \\ f(x) &= \frac{1}{2(x-2)^2} \end{aligned}$$

Division by 0 is not allowed. Set  $x - 2$  equal to 0 to find the restriction.

$$\begin{aligned}x - 2 &= 0 \\x &= 2\end{aligned}$$

7.  $\frac{1}{4}$  or  $\frac{1}{2}$  Solve for  $m$ .

$$\begin{aligned}m^2 - 6m + 8 &= 0 \\(m - 4)(m - 2) &= 0 \\m - 4 = 0 \text{ or } m - 2 &= 0 \\m = 4 \text{ or } m &= 2\end{aligned}$$

So  $\frac{1}{m}$  equals  $\frac{1}{4}$  or  $\frac{1}{2}$ .

8. **(A)** To find the equivalent of  $(-4x^3y - xy^3 + 4x) - (7x - 4xy^3 + 2x^2)$ , combine like terms.

$$\begin{aligned}-xy^3 - (-4xy^3) &= 3xy^3 \\4x - 7x &= -3x\end{aligned}$$

There are no other like terms, so arrange the expression as shown in choice (A).

$$-4x^3y + 3xy^3 - 2x^2 - 3x$$

9. **(D)** Expand  $(x - y)^2$ .

$$(x - y)^2 = x^2 - 2xy + y^2$$

It was given that  $x^2 + y^2 = 12$  and  $xy = -13$ . Therefore,  $-2xy = 26$ .

$$(x - y)^2 = 12 + 26 = 38$$

Multiply the equation by 2.

$$2(x - y)^2 = (2)(38) = 76$$

10. **(A)** You are given  $\frac{m^{x^2}}{m^{y^2}} = m^{36}$ . When dividing like terms, subtract the exponents.

$$m^{x^2 - y^2} = m^{36}$$

Since the bases are the same, drop  $m$  to get  $x^2 - y^2 = 36$ . Factor by using the difference of two squares.

$$(x + y)(x - y) = 36$$

It was given in the question that  $x + y = 9$ , so  $x - y$  must equal 4 because  $9 \times 4 = 36$ .

## Drill 2

For multiple-choice questions, solve the problem and pick the correct answer from the provided choices. Each multiple-choice question has only one correct answer. For student-produced response questions, solve each problem and write down your answers separately.

$$\frac{1}{x^2 - 7x - 98}$$

1. What are the restrictions in the fraction above?
  - (A)  $x \neq -12$  and  $x \neq 8$
  - (B)  $x \neq 12$  and  $x \neq 8$
  - (C)  $x \neq 14$  and  $x \neq -7$
  - (D)  $x \neq -14$  and  $x \neq 7$
2. If  $f(x) = 3(x + 4)^2$ , what is the value of  $f(-2)$ ?

- (A) 4
- (B) 12
- (C) 24
- (D) 36

$$\frac{1}{x} + \frac{2}{x-3} = 6$$

3. What are values of  $x$  in the equation above?

- (A) 7, -2
- (B) -7, 2
- (C)  $\frac{7 \pm \sqrt{41}}{4}$
- (D)  $\frac{2 \pm \sqrt{33}}{6}$

4. Which of the following completely factors  $x^4 - 81$ ?

- (A)  $(x^2 - 9)(x^2 + 9)$
- (B)  $(x + 3)(x - 3)(x^2 - 9)$
- (C)  $(x + 3)^2 (x - 3)^2$
- (D)  $(x + 3)(x - 3)(x^2 + 9)$

5. Which expression is equivalent to  $3x^2 - (5x^2 - 9x^2)$ ?

- (A)  $7x^2$
- (B)  $2x^2$
- (C)  $-x^2$
- (D)  $-11x^2$

6. What is the perimeter of a rectangle with a length that measures  $7x^2 + 5x - 3$  and a width that measures  $2x^2 + 5$ ?

- (A)  $4x^2 + 15x - 9$
- (B)  $9x^2 + 5x - 3$
- (C)  $x^2 + 2x - 5$
- (D)  $18x^2 + 10x + 4$

7. A rectangle has an area that measures  $36 \text{ in.}^2$  and has a length and width that measure  $x + 6$  and  $x - 3$ , respectively. What is the perimeter

of the rectangle?

- (A) 24 inches
- (B) 30 inches
- (C) 36 inches
- (D) 42 inches

8. How many real zeros are there in the equation  $f(x) = x^2 + 2x + 8$ ?

- (A) 0
- (B) 1
- (C) 2
- (D) 3

$$\frac{(x^2 - 5x + 6)(x^2 - 9)}{(x^2 - 6x + 8)}$$

9. Which answer choice expresses the fraction above in simplest form?

- (A)  $\frac{x^2 - 5x + 6}{(x + 3)^2(x - 3)}$
- (B)  $\frac{x - 4}{(x + 3)^2(x - 3)}$
- (C)  $\frac{x - 2}{(x - 3)^2(x + 3)}$
- (D) 1

10. What polynomial must be subtracted from  $5x^2 - 9x + 3$  to get  $-2x^2 - 13x + 5$ ?

- (A)  $12x^2 + 12x - 4$
- (B)  $12x^2 + 13x + 1$
- (C)  $3x^2 - 22x + 8$
- (D)  $7x^2 + 4x - 2$

### Answer Explanations

1. **(C)** The denominator of a fraction cannot equal 0 because dividing by 0 is undefined. Factor the denominator to find values that make the denominator equal 0.

$$x^2 - 7x - 98 = 0$$

$$(x + 7)(x - 14) = 0$$

Set each pair of parentheses equal to 0 and solve.

$$x + 7 = 0 \qquad x - 14 = 0$$

$$x = -7 \qquad x = 14$$

When  $x = -7$  or  $x = 14$ , the denominator equals 0 and the fraction is undefined.

2. **(B)** Replace  $x$  with  $-2$  in  $f(x) = 3(x + 4)^2$ .

$$f(-2) = 3(-2 + 4)^2 = (3)(4) = 12$$

3. **(C)** The common denominator of  $\frac{1}{x} + \frac{2}{x-3} = 6$  is  $x(x-3)$ . Multiply both sides of the equation by this value.

$$x(x-3)\left(\frac{1}{x} + \frac{2}{x-3} = 6\right)$$

$$x - 3 + 2x = 6(x)(x-3)$$

$$3x - 3 = 6x^2 - 18x$$

$$0 = 6x^2 - 21x + 3$$

Simplify the equation by dividing each term by 3.

$$(0 = 6x^2 - 21x + 3) \div 3 =$$

$$0 = 2x^2 - 7x + 1$$

The simplified equation does not factor, so solve by using the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 2 \quad b = -7 \quad c = 1$$

$$\frac{7 \pm \sqrt{(-7)^2 - 4(2)(1)}}{(2)(2)}$$

$$\frac{7 \pm \sqrt{41}}{4}$$

4. **(D)** Use the model for factoring the difference of two squares.

$$a^2 - b^2 = (a + b)(a - b)$$

$$x^4 - 81 = (x^2 - 9)(x^2 + 9)$$

The expression  $x^2 + 9$  is the sum of two squares and does not factor. However,  $x^2 - 9$  is the difference of two squares and can be factored further.

$$(x^2 + 9)(x^2 - 9) = (x^2 + 9)(x + 3)(x - 3)$$

5. **(A)** Use the order of operations to simplify  $3x^2 - (5x^2 - 9x^2)$ .

$$3x^2 - (5x^2 - 9x^2)$$

$$3x^2 - (-4x^2)$$

$$3x^2 + 4x^2 = 7x^2$$

6. **(D)** Find the perimeter of the rectangle by using the formula  $P = 2l + 2w$  in which  $P$  is the perimeter,  $l$  is the length, and  $w$  is the width.

$$2(7x^2 + 5x - 3) + 2(2x^2 + 5)$$

$$14x^2 + 10x - 6 + 4x^2 + 10$$

$$18x^2 + 10x + 4$$

7. **(B)** The area of a rectangle is found by using the formula  $A = lw$  in which  $A$  is the area,  $w$  is the width, and  $l$  is the length.

$$\begin{aligned}
 (x + 6)(x - 3) &= 36 \\
 x^2 + 3x - 18 &= 36 \\
 x^2 + 3x - 54 &= 0 \\
 (x + 9)(x - 6) &= 0 \\
 x + 9 = 0 \quad x - 6 = 0 \\
 x = -9 \quad x = 6
 \end{aligned}$$

Disregard  $x = -9$  because distance cannot be negative. Substitute 6 for  $x$  in the perimeter formula.

$$\begin{aligned}
 P &= 2l + 2w \\
 P &= 2(6 + 6) + 2(6 - 3) \\
 P &= 24 + 6 \\
 P &= 30
 \end{aligned}$$

8. **(A)** Use the discriminant to discern the number of real solutions.

If  $b^2 - 4ac > 0$ , there are 2 real solutions.

If  $b^2 - 4ac = 0$ , there is 1 real solution.

If  $b^2 - 4ac < 0$ , there are no real solutions.

$$\begin{aligned}
 f(x) &= x^2 + 2x + 8 \\
 a = 1 \quad b = 2 \quad c = 8 \\
 (2)^2 - 4(1)(8) &= -28
 \end{aligned}$$

Since  $-28 < 0$ , there are no real solutions.

9. **(C)** Factor the expressions in the numerator and in the denominator.

$$\frac{(x^2 - 5x + 6)(x^2 - 9)}{(x^2 - 6x + 8)} = \frac{(x - 2)(x - 3)(x + 3)(x - 3)}{(x - 4)(x - 2)}$$

Now divide identical factors in the numerator and denominator.

$$\frac{(x - 2)(x - 3)(x + 3)(x - 3)}{(x - 4)(x - 2)} = \frac{(x - 3)^2(x + 3)}{x - 4}$$

10. **(D)** What number must be subtracted from 10 to get 4? You subtract the difference, 4, from 10. Similarly, subtract  $-2x^2 - 13x + 5$  from  $5x^2 - 9x + 3$ .

$$(5x^2 - 9x + 3) - (-2x^2 - 13x + 5) = 7x^2 + 4x - 2$$

## Exponents and Roots Drill

For multiple-choice questions, solve the problem and pick the correct answer from the provided choices. Each multiple-choice question has only one correct answer. For student-produced response questions, solve each problem and write down your answers separately.

1. If  $\frac{3}{a^2} = 64$ , what is the value of  $\sqrt{a}$ ?
- (A) 2  
(B)  $2\sqrt{2}$   
(C) 4  
(D) 8
2. Which of the following is equal to  $\left(\frac{1}{3}\right)^2$ ?
- (A)  $\sqrt{\frac{1}{6}}$   
(B)  $\sqrt[3]{m^2}$   
(C)  $\sqrt{3^m}$   
(D)  $m\sqrt{m}$

$$2x^{\frac{2}{3}} = 6^{\frac{1}{2}}$$

3. In the equation above, what is the value of  $x$ ?
- (A)  $\frac{27}{8}$

- (B)  $\left(\frac{27}{8}\right)^{\frac{1}{4}}$
- (C)  $\left(\frac{8}{27}\right)^{\frac{1}{4}}$
- (D)  $\frac{16}{27}$

4. Which of the following is equivalent to  $25^{\frac{3}{4}}$ ?

- (A)  $\sqrt[3]{25}$
- (B)  $5\sqrt{5}$
- (C)  $5\sqrt[3]{25}$
- (D) 37.5

5. The square root of  $x$  varies inversely with  $y$ . When  $x = 196$ ,  $y = -\frac{1}{2}$ . What is the value of  $x$  when  $y = -3$ ?

- (A) 3
- (B)  $\frac{49}{9}$
- (B)  $\frac{49}{9}$
- (D) 49

$$x^3 - 6x^2 + 2x - 12 = 0$$

6. For what real value of  $x$  is the equation above true?

7. If  $r = 3\sqrt{3x}$  and  $2r = \sqrt{3x}$ , what is the value of  $x$ ?

8. If  $12x - 3y = 5$ , what is the value of  $\frac{16^{3x}}{8^y}$ ?

- (A) 4
- (B) 16
- (C) 32
- (D) 64

$$\left(\frac{1}{3}\right)^{x-2} = (27)^{3-x}$$

9. Given the equation above, what is the value of  $2x + 11$ ?

10. If  $\frac{y}{x^4} = 81$  for positive integers  $x$  and  $y$ , what is one possible value of  $y$ ?

### Answer Explanations

1. **(C)** Raise each side of the equation to the  $\frac{2}{3}$  power to isolate  $a$ .

$$\begin{aligned}\left(\frac{3}{2}\right)^{\frac{2}{3}} &= 64^{\frac{2}{3}} \\ a &= 16 \\ \sqrt{16} &= 4\end{aligned}$$

2. **(B)** When raising an exponent to another, multiply the exponents.

$$\left(m^{\frac{1}{3}}\right)^2 = m^{\frac{2}{3}}$$

Rational exponents can be expressed as radicals: the numerator is the exponent, and the denominator is the root.

$$m^{\frac{2}{3}} = \sqrt[3]{m^2}$$

The root, 3, exceeds the exponent, 2, so  $\sqrt[3]{m^2}$  is simplified to its lowest term.

3. **(B)** Begin isolating  $x$  by raising both sides of the equation to the sixth power.

$$\begin{aligned}2x^{\frac{2}{3}} &= 6^{\frac{1}{2}} \\ \left(2x^{\frac{2}{3}}\right)^6 &= \left(6^{\frac{1}{2}}\right)^6 \\ 64x^4 &= 6^3 \\ 64x^4 &= 216\end{aligned}$$

Divide both sides of the equation by 64.

$$\frac{64x^4}{64} = \frac{216}{64}$$

$$x^4 = \frac{27}{8}$$

Raise both sides of the equation to the  $\frac{1}{4}$ th power.

$$(x^4)^{\frac{1}{4}} = \left(\frac{27}{8}\right)^{\frac{1}{4}}$$

$$x = \left(\frac{27}{8}\right)^{\frac{1}{4}}$$

4. **(B)** Transform  $25^{\frac{3}{4}}$  to  $(5^2)^{\frac{3}{4}}$ , which equals  $5^{\frac{6}{4}} = 5^{\frac{3}{2}}$ . When a number is raised to a rational power, the numerator serves as the power of the number and the denominator as its root. Therefore, raise 5 to the third power and find the square root of that quantity.

$$\sqrt{5^3} = \sqrt{125} = 5\sqrt{5}$$

Choices (A) and (C) are incorrect because  $25^{\frac{3}{4}}$  transforms to a square root, not a cube root. Choice (D) is incorrect because it multiplies 25 by  $\frac{3}{2}$  rather than raising it to the  $\frac{3}{4}$  power.

5. **(B)** Inverse variation uses the model  $xy = k$  where  $k$  is a constant. Since the question stipulates that the square root of  $x$  varies inversely with  $y$ , use the model  $\sqrt{x}y = k$ .

$$\sqrt{196} \times -\frac{1}{2} = k$$

$$14 \times -\frac{1}{2} = k$$

$$-7 = k$$

Now input  $y = -3$  into the equation.

$$\sqrt{x} \times -3 = -7$$

$$\sqrt{x} = \frac{7}{3}$$

$$(\sqrt{x})^2 = \left(\frac{7}{3}\right)^2$$

$$x = \frac{49}{9}$$

6. **6** Begin factoring  $x^3 - 6x^2 + 2x - 12 = 0$  by grouping.

$$(x^3 - 6x^2) + (2x - 12) = 0$$

Extract the greatest common factor from the expressions in the parentheses.

$$x^2(x - 6) + 2(x - 6) = 0$$

Place  $x^2$  and 2 into its own parentheses.

$$(x^2 + 2)(x - 6) = 0$$

Set each of the parentheses equal to 0.

$$x^2 + 2 = 0 \text{ or } x - 6 = 0$$

$$x^2 = -2 \text{ or } x = 6$$

$x^2 = -2$  becomes  $x = \pm i\sqrt{2}$ , neither of which is a real number. Therefore, the real solution to  $x^3 - 6x^2 + 2x - 12 = 0$  is  $x = 6$ .

7. **36** The question says that  $r = 3\sqrt{3}$  and  $2r = \sqrt{3x}$ . Therefore,  $2r$  also equals  $(2)(3\sqrt{3})(2)(3\sqrt{3})$  or  $6\sqrt{3}$ . Set  $6\sqrt{3}$  equal to  $\sqrt{3x}$  and solve for  $x$ .

Remove the radicals by squaring both sides.

$$6\sqrt{3} = \sqrt{3x}$$

$$(6\sqrt{3})^2 = (\sqrt{3x})^2$$

$$108 = 3x$$

$$36 = x$$

8. **(C)** Convert the numerator and the denominator to base 2.

$$\frac{16^{3x}}{8^y} = \frac{(2^4)^{3x}}{(2^3)^y} = \frac{2^{12x}}{2^{3y}}$$

When dividing terms with the same base, subtract the exponents.

$$\frac{2^{12x}}{2^{3y}} = 2^{12x-3y}$$

Since  $12x - 3y = 5$ , then  $2^{12x-3y} = 2^5 = 32$ .

9. **18** Solve for  $x$  by changing each base to 3.

$$\begin{aligned}\left(\frac{1}{3}\right)^{x-2} &= (27)^{3-x} \\ (3^{-1})^{x-2} &= (3^3)^{3-x} \\ (3)^{-x+2} &= (3)^{9-3x} \\ -x + 2 &= 9 - 3x \\ -7 &= -2x \\ \frac{7}{2} &= x \\ \text{or} \\ 3.5 &= x\end{aligned}$$

Substitute 3.5 for  $x$  in  $2x + 11$ .

$$(2)(3.5) + 11 = 18$$

10. **1, 2, 4, 8, or 16** The equation  $x^{\frac{y}{4}} = 81$ , where  $x$  and  $y$  are positive integers, can be written as follows.

$$\begin{aligned}3^4 &= 81 \\9^2 &= 81 \\81^1 &= 81 \\(81^2)^{\frac{1}{2}} &= 81 \\(81^4)^{\frac{1}{4}} &= 81\end{aligned}$$

If  $x^{\frac{y}{4}} = 81$ , and if  $x$  and  $y$  are positive integers, then  $\frac{y}{4}$  can equal 4, 2, 1,  $\frac{1}{2}$ , or  $\frac{1}{4}$ . Therefore, the value of  $y$  can be 16, 8, 4, 2, or 1.

## Solving Advanced Equations

### Drill 1

For multiple-choice questions, solve the problem and pick the correct answer from the provided choices. Each multiple-choice question has only one correct answer. For student-produced response questions, solve each problem and write down your answers separately.

$$\sqrt{n-b} = n-2$$

- If  $b = -4$ , what is the solution set to the equation above?
  - $\{-5\}$
  - $\{0\}$
  - $\{5\}$
  - $\{0, 5\}$
- What are the solutions to  $6x^2 + 12x - 15 = 0$ ?
  - $\frac{2 \pm \sqrt{14}}{2}$
  - $\frac{2 \pm \sqrt{14}}{2}$

(C)  $\frac{-2 \pm \sqrt{14}}{2}$

(D)  $-2 \pm 4\sqrt{2}$

3. If  $(x + n)^2 = x^2 + 19x + n^2$ , what is the value of  $n^2$ ?

(A) 361

(B)  $\frac{361}{2}$

(C)  $\frac{361}{4}$

(D) 38

4. What are the solutions to  $2x^2 + 8x - 12 = 0$ ?

(A)  $x = -4 \pm 3\sqrt{10}$

(B)  $x = -3 + \frac{\sqrt{10}}{10}$

(C)  $x = -2 \pm \sqrt{10}$

(D)  $x = 2 \pm \sqrt{10}$

$$x - 2 = \frac{4}{x - 2}$$

5. Which of the following is the solution to the equation above?

(A) -4

(B) 0

(C) 0, 4

(D) 2, -2

6. The theory of special relativity suggests that an observer moving at great speed experiences the flow of time much more slowly than an observer on Earth. The following formula is used to calculate what is known as time dilation.

$$F(t) = \frac{t}{\sqrt{1 - v^2}}$$

In the formula,  $t$  represents some unit of time and  $v$  represents velocity as some fraction of light-speed. For example, an observer counts 1 second in a rocket ship traveling at 60% of the speed of light while an

observer on Earth measures 1.25 seconds. If an observer traveling at 80% of the speed of light counts off 1 second, how many seconds will be counted by an observer on Earth?

7. The theory of special relativity suggests that an observer moving at great speed experiences the flow of time much more slowly than an observer on Earth. The following formula is used to calculate what is known as time dilation.

$$F(t) = \frac{t}{\sqrt{1 - v^2}}$$

In the formula,  $t$  represents some unit of time and  $v$  represents velocity as some fraction of light-speed. For example, an observer counts 1 second in a rocket ship traveling at 60% of the speed of light while an observer on Earth measures 1.25 seconds. An observer on Earth measures 2 seconds, while an observer traveling at a fraction of the speed of light measures 1 second. At what fraction of the speed of light was the moving observer traveling?

8.  $C(x) = x^3 + 5x^2 + 5x - 2$

In the function above, which of the following can be assumed?

- (A)  $x + 3$  is a factor
  - (B)  $x + 2$  is a factor
  - (C)  $x - 5$  is a factor
  - (D)  $x + 4$  is a factor
9.  $(ax - 7)(bx + 5) = 6x^2 + cx - 35$ . If  $ab = 6$  and  $a - b = -1$ , what are two possible values of  $c$ ?
- (A)  $c = 2$  or  $c = -4$
  - (B)  $c = -11$  or  $c = -1$
  - (C)  $c = -2$  or  $c = 4$
  - (D)  $c = 12$  or  $c = -5$

$$x^3 + x^2 - 210x = 0$$

10. What is the sum of the roots in the equation above?

- (A) -14
- (B) -1
- (C) 1
- (D) 15

### Answer Explanations

1. (C) Square both sides of the equation to get rid of the radical.

$$\begin{aligned}\sqrt{n - (-4)} &= n - 2 \\ \sqrt{n - (-4)}^2 &= (n - 2)^2 \\ n + 4 &= n^2 - 4n + 4 \\ 0 &= n^2 - 5n \\ 0 &= n(n - 5) \\ n = 0 \text{ or } n - 5 &= 0 \\ n = 0 \text{ or } n &= 5\end{aligned}$$

Check each solution in the original equation to ascertain any extraneous solutions.

Check 5:

$$\begin{aligned}\sqrt{5 - (-4)} &= 5 - 2 \\ \sqrt{9} &= 3 \\ 3 &= 3\end{aligned}$$

Check 0:

$$\begin{aligned}\sqrt{0 - (-4)} &= 0 - 2 \\ \sqrt{4} &= -2 \\ 2 &\neq -2\end{aligned}$$

We accept 5 as the solution and reject 0.

2. **(C)** Solve the equation by first dividing all of the terms by 3.

$$(6x^2 + 12x - 15) \div 3 = 0 \div 3$$

$$2x^2 + 4x - 5 = 0$$

The equation cannot be factored, so use the quadratic equation to solve.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 2, \quad b = 4, \quad c = -5$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(2)(-5)}}{2(2)}$$

$$x = \frac{-4 \pm \sqrt{16 + 40}}{4}$$

$$x = \frac{-4 \pm \sqrt{56}}{4} = \frac{-2 \pm \sqrt{14}}{2}$$

3. **(C)** Find the value of  $n^2$  by completing the square in  $x^2 + 19x + n^2$ .  
Multiply 19 by  $\frac{1}{2}$  and square the product.

$$x^2 + 19x + n^2 = x^2 + 19x + \left(\frac{19}{2}\right)^2 = x^2 + 19x + \frac{361}{4}$$

$$n^2 = \frac{361}{4}$$

4. **(C)** Divide both sides of the equation by 2, which is the greatest common factor of  $2x^2 + 8x - 12$ .

$$(2x^2 + 8x - 12) \div 2 = 0 \div 2$$

$$x^2 + 4x - 6 = 0$$

The equation does not factor, so solve by using the quadratic equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 1, \quad b = 4, \quad c = -6$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-6)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{40}}{2}$$

$$x = \frac{-4 \pm 2\sqrt{10}}{2}$$

$$x = 2 \pm \sqrt{10}$$

5. **(C)** Some test takers may find that a plug-and-check strategy is best for them. If you solve the question correctly and quickly, using that method is fine. Here is the math needed to solve this equation. Multiply both sides of the equation by  $x - 2$ .

$$(x - 2)(x - 2) = \left(\frac{4}{x - 2}\right)(x - 2)$$

$$(x - 2)^2 = 4$$

Find the square root of both sides of the equation.

$$\sqrt{(x - 2)^2} = \sqrt{4}$$

$$x - 2 = \pm 2$$

$$x = 4 \quad x = 0$$

Check for extraneous solutions.

$$0 - 2 = \frac{4}{0 - 2}$$

$$-2 = -2$$

$$4 - 2 = \frac{4}{4 - 2}$$

$$2 = 2$$

Neither 4 nor 0 is an extraneous solution. So both 4 and 0 are solutions to the equation.

6. **1.66 or 1.67 or  $\frac{5}{3}$**  Input 80% for  $v$  into the formula of special relativity.

$$F(t) = \frac{t}{\sqrt{1 - v^2}}$$
$$F(1) = \frac{1}{\sqrt{1 - (0.8)^2}} = 1.\overline{66}$$

7. **.866** Input 1 and 2 for  $t$  and  $F(t)$ , respectively, into the special relativity formula.

$$F(t) = \frac{t}{\sqrt{1 - v^2}}$$
$$2 = \frac{1}{\sqrt{1 - v^2}}$$
$$2(\sqrt{1 - v^2}) = 1$$
$$\sqrt{1 - v^2} = 0.5$$
$$(\sqrt{1 - v^2})^2 = (0.5)^2$$
$$1 - v^2 = 0.25$$
$$-v^2 = -0.75$$
$$v = 0.866$$

8. **(B)** If  $x + 2$  is a factor of  $C(x) = x^3 + 5x^2 + 5x - 2$ , the remainder, after dividing, is 0.

$$\begin{array}{r} x^2 + 3x - 1 \\ x + 2 \overline{) x^3 + 5x^2 + 5x - 2} \end{array}$$

Since there is no remainder, we know that  $x + 2$  is factor of  $C(x) = x^3 + 5x^2 + 5x - 2$ . An alternate method to answering this question is to substitute  $-2$  for  $x$  and assess the results.

$$(-2)^3 + 5(-2)^2 + 5(-2) - 2 = 0$$

The result is 0, which indicates  $-2$  is a root of the function and  $x + 2$  is a factor.

9. **(B)**

$$(ax - 7)(bx + 5) = 6x^2 + cx - 35$$

Solve for  $a$  or  $b$  in  $a - b = -1$  and create a system of equations.

$$a - b = -1 \text{ so } a = b - 1$$

Substitute  $b - 1$  for  $a$  in  $ab = 6$ .

$$\begin{aligned}(b - 1)b &= 6 \\ b^2 - b &= 6 \\ b^2 - b - 6 &= 0\end{aligned}$$

Factor and solve for  $b$ .

$$\begin{aligned}(b + 2)(b - 3) &= 0 \\ b + 2 = 0 \text{ or } b - 3 &= 0 \\ b = -2 \text{ or } b &= 3\end{aligned}$$

Find  $a$  when  $b = 3$  or  $b = -2$ .

$$\begin{aligned}ab &= 6 \\ 3a &= 6 \\ a &= 2\end{aligned}$$

When  $b = 3$ ,  $a = 2$ .

$$\begin{aligned}ab &= 6 \\ -2a &= 6 \\ a &= -3\end{aligned}$$

When  $b = -2$ ,  $a = -3$ .

Solve for  $c$  when  $a$  and  $b$  are 2 and 3, respectively.

$$(2x - 7)(3x + 5) = 6x^2 - 11x - 35$$

$$c = -11$$

Solve for  $c$  when  $a$  and  $b$  are  $-3$  and  $-2$ , respectively.

$$(-3x - 7)(-2x + 5) = 6x^2 - x - 35$$

$$c = -1$$

You only have to solve for one of the values of  $c$ . Choice (B) is the only one that has either of the correct solutions to the question.

10. **(B)** Factor the equation.

$$x^3 + x^2 - 210x = 0$$

$$x(x^2 + x - 210) = 0$$

$$x(x + 15)(x - 14) = 0$$

$$x = 0 \quad x = -15 \quad x = 14$$

$$0 - 15 + 14 = -1$$

## Drill 2

For multiple-choice questions, solve the problem and pick the correct answer from the provided choices. Each multiple-choice question has only one correct answer. For student-produced response questions, solve each problem and write down your answers separately.

1. For what value(s) of  $x$  is the following expression undefined?

$$\frac{-5x^2y - 4xy}{6x^2 - 13x - 5}$$

- (A)  $-1, 5$

- (B) 5
- (C)  $-\frac{1}{3}, \frac{5}{2}$
- (D)  $\frac{1}{3}, \frac{3}{2}$

$$\sqrt{2x} = x - 4$$

2. What value(s) of  $x$  satisfy the equation above?

- I. 8
- II. 2
- III. Both 8 and 2

- (A) I only
- (B) II only
- (C) I and II
- (D) Neither value satisfies the equation.

3. Which of the following is a step in solving the equation  $\sqrt{x-7} = x-7$ ?

- (A)  $x - 7 = x - 7$
- (B)  $2x = 14$
- (C)  $(x - 8)(x - 7) = 0$
- (D)  $(x + 7)(x - 6) = 0$

4. Simplify  $-2\left(\frac{x^2 - 8x - 180}{x + 10}\right)$ , if  $x \neq -10$ .

- (A)  $x - 18$
- (B)  $-2x + 36$
- (C)  $-2x^2 + x$
- (D)  $x + 10$

5. An annuity is a fixed income stream paid out over an agreed upon period of time. A \$1,000,000 insurance payment is an annuity if it is paid out over a period of time in fixed installments. The insurance company may offer the payee a lump sum payout called the present

value of the annuity. The formula to calculate the present value of an annuity is as follows.

$$P = A\left(1 + \frac{r}{n}\right)^{-nt}$$

where:

$P$  = the present value of the payout

$A$  = the dollars that would have been received over time

$t$  = time in years

$r$  = the interest rate expressed as a decimal

$n$  = compounding periods each year

Jorge and his brothers have inherited a pension plan from their grandfather. The plan will pay each grandchild \$10,000 over the course of a 10-year period. Jorge chooses to accept the present value of the inheritance at 8% compounded monthly. How much should he expect his payout to be? (Round your answer to the nearest dollar.)

6. An annuity is a fixed income stream paid out over an agreed upon period of time. A \$1,000,000 insurance payment is an annuity if it is paid out over a period of time in fixed installments. The insurance company may offer the payee a lump sum payout called the present value of the annuity. The formula to calculate the present value of an annuity is as follows.

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where:

$P$  = the present value of the payout

$A$  = the dollars that would have been received over time

$t$  = time in years

$r$  = the interest rate expressed as a decimal

$n$  = compounding periods each year

Samantha received a cash annuity as a gift for college graduation. The annuity would pay an interest rate of 3.5% over a 5-year period

compounded quarterly. Samantha elected to receive the present value of the annuity, which totaled \$1,621. To the nearest dollar, what would have been the payout of the annuity if she chose to accept the 5-year payout terms?

7. What is the value of  $x$  if  $x = \frac{w^2 + 2w + 1}{\frac{w + 1}{3}}$ ?
- (A)  $\frac{3}{w + 1}$   
(B)  $3(w + 1)$   
(C)  $\frac{1}{3w + 1}$   
(D)  $\frac{1}{3 + w}$
8. The points  $(2, 7)$  and  $(-3, 37)$  lie on the graph of  $y = ax^2 - 4x + 7$ . What is the value of  $a$ ?
9. What is the sum of all values of  $x$  that satisfy  $4x^2 - 2x - 6 = 0$ ?
- (A)  $-\frac{\sqrt{2}}{2}$   
(B)  $-\frac{1}{2}$   
(C)  $\frac{1}{2}$   
(D)  $2\sqrt{2}$

$$x^3 - 6x^2 + 2x - 12 = 0$$

10. What real value of  $x$  makes the above equation true?

### Answer Explanations

1. **(C)** Division by 0 is undefined. Set  $6x^2 - 13x - 5$  equal to 0 and solve.

$$6x^2 - 13x - 5 = 0$$

Solve by factoring.

$$(3x + 1)(2x - 5) = 0$$

Set each of the parentheses equal to 0, and solve for the variable.

$$3x + 1 = 0 \text{ or } 2x - 5 = 0$$

$$3x = -1 \text{ or } 2x = 5$$

$$x = -\frac{1}{3} \text{ or } x = \frac{5}{2}$$

The expression  $\frac{-5x^2y - 4xy}{6x^2 - 13x - 5}$  is undefined when  $x = -\frac{1}{3}$  and when  $x = \frac{5}{2}$ .

2. **(A)** Square both sides of the equation to eliminate the radical.

$$(\sqrt{2x})^2 = (x - 4)^2$$

$$2x = x^2 - 8x + 16$$

Set the equation equal to 0 and factor.

$$2x = x^2 - 8x + 16$$

$$x^2 - 10x + 16 = 0$$

$$(x - 8)(x - 2) = 0$$

Solve for  $x$ .

$$x - 8 = 0 \text{ or } x - 2 = 0$$

$$x = 8 \text{ or } x = 2$$

Check both values to eliminate extraneous answers.

$$\sqrt{2 \times 8} = 8 - 4$$

$$4 = 4$$

$$\sqrt{2 \times 2} = 2 - 4$$

$$2 \neq -2$$

Only 8 is a solution because 2 is an extraneous solution.

3. **(C)** Square both sides of the equation to remove the radical.

$$\begin{aligned}
 (x - 7)^2 &= (x - 7)^2 \\
 x - 7 &= x^2 - 14x + 49 \\
 x &= x^2 - 14x + 56 \\
 x^2 - 15x + 56 &= 0 \\
 (x - 7)(x - 8) &= 0
 \end{aligned}$$

This last step reflects answer choice (C).

4. **(B)** Factor and cancel as needed.

$$\begin{aligned}
 -2\left(\frac{x^2 - 8x - 180}{x + 10}\right) &= \\
 -2\left(\frac{(x + 10)(x - 18)}{x + 10}\right) &= \\
 -2(x - 18) &= -2x + 36
 \end{aligned}$$

5. **4,505** Use the present value formula and input the known information.

$$\begin{aligned}
 P &= A\left(1 + \frac{r}{n}\right)^{-nt} \\
 P &= 10,000\left(1 + \frac{0.08}{12}\right)^{-(12)(10)} = 4,505
 \end{aligned}$$

Jorge can expect to receive \$4,505 as the present value of his \$10,000 ten-year annuity.

6. **1,930** Input the known information into the present value formula and solve for A.

$$\begin{aligned}
 P &= A\left(1 + \frac{r}{n}\right)^{-nt} \\
 1,621 &= A\left(1 + \frac{0.35}{4}\right)^{-(4)(5)} \\
 1,621 &= 0.84A \\
 1,930 &= A
 \end{aligned}$$

Samantha would have earned \$1,930 if she had elected to receive her gift as an annuity.

7. **(B)** Factor the numerator of the complex fraction.

$$\frac{\frac{w^2 + 2w + 1}{w + 1}}{\frac{3}{w + 1}} = \frac{(w + 1)(w + 1)}{\frac{3}{w + 1}}$$

When dividing a complex fraction, invert the denominator and multiply that quantity with the numerator.

$$\frac{(w + 1)(w + 1)}{\frac{3}{w + 1}} = (w + 1)(w + 1)\left(\frac{3}{w + 1}\right)$$

Cross-divide as needed.

$$= (w + 1)(w + 1)\left(\frac{3}{w + 1}\right) = 3(w + 1)$$

8. **2** Substitute either (2, 7) or (-3, 37) for  $x$  and  $y$ , and solve for  $a$ .

$$7 = (a)(2)^2 - (4)(2) + 7$$

$$7 = 4a - 8 + 7$$

$$8 = 4a$$

$$2 = a$$

9. **(C)** This problem can be solved by factoring  $4x^2 - 2x - 6 = 0$  or by using the quadratic equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 4, b = -2, c = -6$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(4)(-6)}}{2(4)}$$

$$x = \frac{2 \pm \sqrt{4 - (-96)}}{8}$$

$$x = \frac{2 \pm 10}{8}$$

$$x = \frac{3}{2}, x = -1$$

The sum of the roots,  $-1$  and  $\frac{3}{2}$ , is  $\frac{1}{2}$ .

10. 6 Factor by grouping.

$$\begin{aligned}x^3 - 6x^2 + 2x - 12 &= 0 \\(x^3 - 6x^2) + (2x - 12) &= 0 \\x^2(x - 6) + 2(x - 6) &= 0 \\(x^2 + 2)(x - 6) &= 0 \\x^2 + 2 = 0 \text{ or } x - 6 = 0 \\x = \pm i\sqrt{2} \text{ or } x = 6\end{aligned}$$

Of the three solutions, only 6 is a real solution to the equation.

## Zeros, Parabolas, and Polynomial Graphing Drill

For multiple-choice questions, solve the problem and pick the correct answer from the provided choices. Each multiple-choice question has only one correct answer. For student-produced response questions, solve each problem and write down your answers separately.

- Which of the following equations has the same  $x$ - and  $y$ -coordinates for the vertex as does  $y = 3(x - 4)(x + 6)$ ?
  - $y = x^2 - 8x + 15$
  - $y = 3(x^2 + 4x - 6)$
  - $y = 3(x + 1)^2 - 75$
  - $y = 3x^2 + 6x - 10$
- The function  $g$  is defined by  $g(x) = 2x^2 + 15x - 8$ . The  $x$ -coordinate of the vertex of the graph is contained within which of the following intervals?
  - $-\frac{1}{2} < x < 8$

- (B)  $-2 < x < 2$
- (C)  $-3.65 < x < -2$
- (D)  $-4 < x < 6$

$$y = x^2 + 2x - 24$$

3. The equation above represents a parabola in the  $xy$ -plane. Which of the following equations shows the  $x$ -intercepts of the parabola as constants or coefficients?

- (A)  $y - 24 = x(x + 2)$
- (B)  $y = (x + 1)^2 - 25$
- (C)  $y = (x + 6)(x - 4)$
- (D)  $y = x(x + 2) - 24$

4. Which of the following equations has only one  $x$ -intercept in the  $xy$ -plane?

- (A)  $x^2 + 2x - 48 = y$
- (B)  $x^2 + 11x + 30 = y$
- (C)  $x^2 - 6x + 8 = y$
- (D)  $x^2 + 6x + 9 = y$

5. A furniture manufacturer of office chairs uses the following profit function.

$$P(x) = -0.08x^2 + 23.1x + 500$$

where  $x$  represents the number of chairs that are manufactured and  $P(x)$  is profit in dollars. What is the most profitable number of chairs to manufacture? (Round your answer to the nearest whole number.)

6. Which of the following is true about the graph of the equation  $y = 4(x - 3)^2 + 13$ ?
- (A) The graph is wider than the graph of  $y = (x - 3)^2 + 13$ .
  - (B) It has no real roots.
  - (C) Its minimum value is  $-3$ .

(D) Its roots are  $x = 3$  and  $x = -13$ .

$$h = -4.9t^2 + 32t$$

7. The equation above shows the height of a ball thrown upward from ground level with an initial velocity of 32 meters per second.

After approximately how many seconds will the ball hit the ground?

- (A) 5.6
- (B) 6.5
- (C) 7.4
- (D) 8.2

$$h = -4.9t^2 + 32t$$

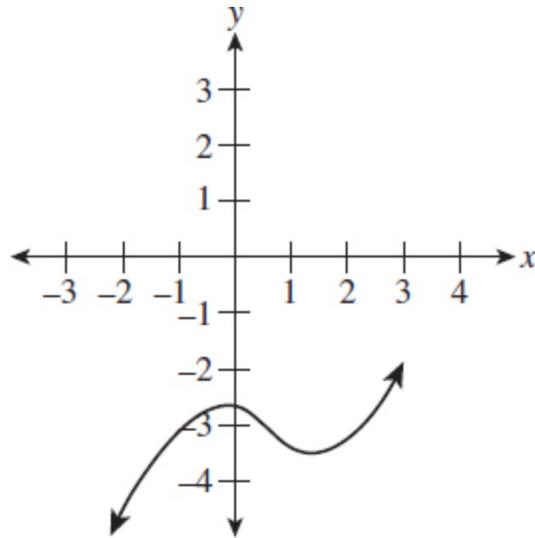
8. The equation above shows the height of a ball thrown upward from ground level with an initial velocity of 32 meters per second. After how many seconds will the ball be 18 meters in the air?

- (A) 2.2
- (B) 4.3
- (C) 0.6 and 5.9
- (D) 0.6 or 6.3

9. Which of the following is an equivalent form of the function  $h$  such that the minimum value of  $h$  appears as a constant or coefficient?

$$h(x) = (x - 8)(x + 2)$$

- (A)  $h(x) = x^2 - 6x - 16$
- (B)  $h(x) = x^2 + 6x - 16$
- (C)  $h(x) = (x - 3)^2 - 25$
- (D)  $h(x) = (x + 3)^2 - 25$



10. Consider the graph of the equation above. How many times would it intersect  $f(x) = -3$ ?

### Answer Explanations

1. **(C)** Find the coordinates of the vertex of  $3(x - 4)(x + 6)$  by multiplying.

$$y = 3(x - 4)(x + 6)$$

$$y = 3(x^2 + 2x - 24)$$

Begin to transform the equation from standard form to graphing form. First, move  $-24$  to the right of the parentheses, remembering to first multiply it by 3.

$$y = 3(x^2 + 2x) - (24)(3)$$

$$y = 3(x^2 + 2x) - 72$$

Complete the square by squaring  $\frac{b}{2}$ . Remember to multiply that value by 3.

$$y = 3(x^2 + 2x + 1) - 72 - (3 \times 1)$$

$$y = 3(x^2 + 2x + 1) - 72 - 3$$

$$y = 3(x + 1)^2 - 75$$

The equation is now in graphing form. The vertex is  $(-1, -75)$ .

2. **(D)** The  $x$ -coordinate of the vertex can be found using the formula  $x = \frac{-b}{2a}$ . Find the values of  $a$  and  $b$ .

$$\begin{aligned}g(x) &= 2x^2 + 15x - 8 \\a &= 2, b = 15 \\x &= \frac{-b}{2a} = \frac{-15}{4} = -3.75\end{aligned}$$

Answer choice **(D)**,  $-4 < x < 6$ , is the only interval among the answer choices that contains  $x = -3.75$ .

3. **(C)** The factored form of  $y = x^2 + 2x - 24$  is  $y = (x + 6)(x - 4)$ . To find the  $x$ -intercepts, let  $y = 0$ .

$$\begin{aligned}0 &= (x + 6)(x - 4) \\x + 6 &= 0 & x - 4 &= 0 \\x &= -6 & x &= 4\end{aligned}$$

The factored form of  $y = x^2 + 2x - 24$  shows the  $x$ -intercepts of the parabola as constants.

4. **(D)** Find the  $x$ -intercept(s) by letting  $y = 0$ .

Choice **(A)**:

$$\begin{aligned}x^2 + 2x - 48 &= 0 \\(x + 8)(x - 6) &= 0 \\x &= -8 \text{ and } x = 6\end{aligned}$$

There are two  $x$ -intercepts.

Choice **(B)**:

$$\begin{aligned}x^2 + 11x + 30 &= 0 \\(x + 6)(x + 5) &= 0 \\x &= -6 \text{ and } x = -5\end{aligned}$$

There are two  $x$ -intercepts.

Choice (C):

$$\begin{aligned}x^2 - 6x + 8 &= 0 \\(x - 4)(x - 2) &= 0 \\x = 4 \text{ and } x = 2\end{aligned}$$

There are two  $x$ -intercepts.

Choice (D):

$$\begin{aligned}x^2 + 6x + 9 &= 0 \\(x + 3)^2 &= 0 \\x &= -3\end{aligned}$$

There is only one  $x$ -intercept.

5. **144** The profit function,  $P(x) = -0.08x^2 + 23.1x + 500$ , is a parabola that opens down. The vertex is a maximum with the  $x$ -coordinate representing the most profitable number of chairs to manufacture. Find the  $x$ -coordinate by using the formula for finding the axis of symmetry,  $x = \frac{-b}{2a}$ .

$$\begin{aligned}a &= -0.08, b = 23.1 \\x &= \frac{-23.1}{2(-0.08)} = 144.375 \approx 144\end{aligned}$$

6. **(B)** One way to solve this problem is to let  $y = 0$ . Doing so will let us find the roots of the equation.

$$\begin{aligned}0 &= 4(x - 3)^2 + 13 \\-13 &= 4(x - 3)^2 \\-\frac{13}{4} &= (x - 3)^2 \\\pm \frac{\sqrt{-13}}{4} &= x - 3 \\3 \pm \frac{i\sqrt{13}}{2} &= x\end{aligned}$$

7. **(B)** The ball hits the ground when the height is 0 feet. Let  $h$  equal 0 and solve for  $t$ .

$$\begin{aligned}0 &= -4.9t^2 + 32t \\0 &= t(-4.9t + 32) \\t = 0 \text{ or } -4.9t + 32 &= 0 \\t = 0 \text{ or } t &= 6.5\end{aligned}$$

The value of  $t = 0$  represents the time before the ball was thrown.

8. **(C)** Replace  $h$  with 18 as the height the ball attains. Use the formula  $h = -4.9t^2 + 32t$ .

$$18 = -4.9t^2 + 32t$$

Set the equation equal to 0 and solve for  $t$ .

$$-4.9t^2 + 32t - 18 = 0$$

The equation does not factor, so use the quadratic formula  $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ .

$$a = -4.9, b = 32, c = -18$$

After using the quadratic formula, we arrive at two different answers, 0.6 and 5.9. We get two different answers because after 0.6 seconds the ball attains a height of 18 meters and then rises higher. As the ball returns to Earth, it reaches 18 meters high again, 5.9 seconds after it was thrown into the air.

9. **(C)** Transform  $h(x) = (x - 8)(x + 2)$  to graphing form by multiplying the expressions within parentheses.

$$h(x) = (x - 8)(x + 2) = x^2 - 6x - 16$$

Move  $-16$  to the far right of the equation as you complete the square. Remember that as you add a value to complete the square, that same value must be subtracted from  $-16$ .

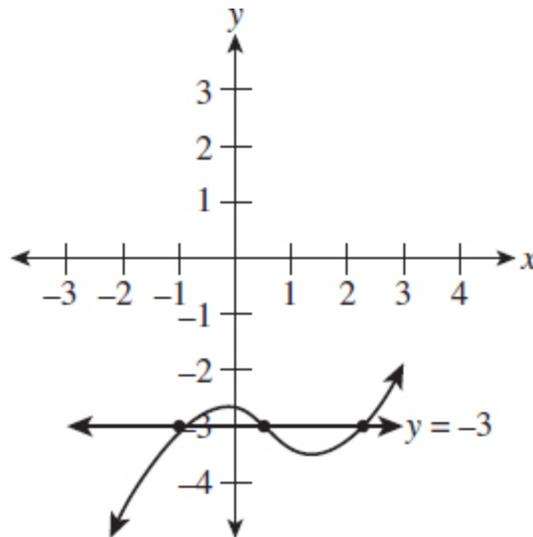
$$h(x) = x^2 - 6x - 16$$

$$h(x) = x^2 - 6x + 9 - 16 - 9$$

$$h(x) = (x - 3)^2 - 25$$

The vertex of the parabola is  $(3, -25)$ , and the minimum value is  $-25$ .

10. **3** Remember that  $f(x) = -3$  can be expressed as  $y = -3$ . Draw the graph of  $y = -3$  on the graph of the cubic equation.



By inspection, the number of intersections is 3.

## Function Interpretation and Manipulation

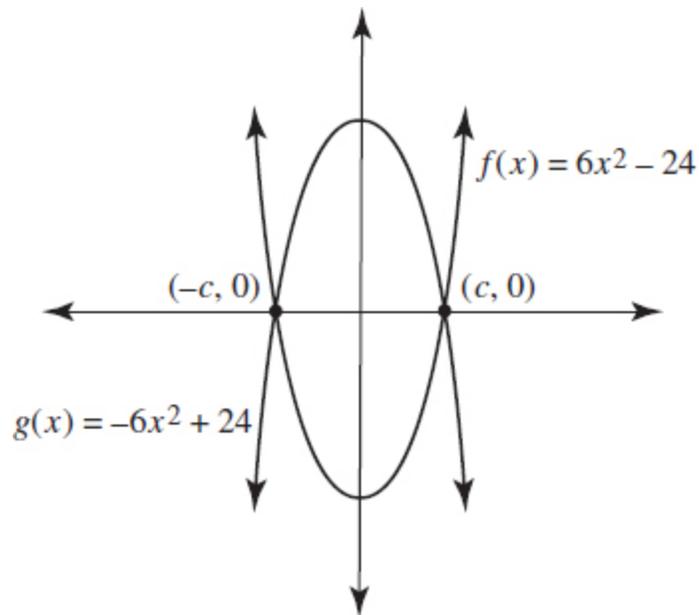
### Drill 1

For multiple-choice questions, solve the problem and pick the correct answer from the provided choices. Each multiple-choice question has only one correct answer. For student-

produced response questions, solve each problem and write down your answers separately.

1. If  $m = 2x^3 + 3x - 11$  and  $n = 4x^2 + 5x - 7$ , what is  $3n - m$  in terms of  $x$ ?

- (A)  $-2x^3 - 4x^2 - 2x - 18$
- (B)  $-(2x^3 + 3x^2 - 5x - 6)$
- (C)  $2x^3 - 12x^2 - 12x + 10$
- (D)  $-2x^3 + 12x^2 + 12x - 10$



Note: Figure not drawn to scale

2. The functions  $f$  and  $g$ , defined by  $f(x) = 6x^2 - 24$  and  $g(x) = -6x^2 + 24$ , are graphed in the  $xy$ -plane above. The graphs intersect at the points  $(c, 0)$  and  $(-c, 0)$ . What is the value of  $c$ ?

- (A)  $-2$
- (B)  $0$
- (C)  $2$
- (D)  $4$

3. Conner needs to plan a production budget for a computer firm. In order to maximize profit, he needs to buy  $m$  computer frames at \$18.75 and  $r$  keyboards at \$13.25 each. If the cost of this transaction must be less than \$139.00, which of the following inequalities represents Conner's model for profit maximization?

- (A)  $18.75m + 13.25r < 139$
- (B)  $18.75m + 13.25r \leq 139$
- (C)  $18.75r + 13.25m < 139$
- (D)  $18.75r + 13.25m \leq 139$

4. Californians are very concerned about the continued drought conditions in their state. They are therefore curious about the price and quality of different water plans. A certain popular water plan charges a service fee of \$31.50 per month. Each gallon of water used costs \$0.17 per gallon plus a local tax of 7.3% (the monthly service charge is tax exempt). Which of the following functions models the monthly cost of the plan?

- (A)  $f(x) = (0.17x + 0.073)(31.5)$
- (B)  $f(x) = 31.5 + (0.073x)(0.17)$
- (C)  $f(x) = 31.5 + (1.073)(0.17 + x)$
- (D)  $f(x) = 31.5 + (1.073)(0.17x)$

5. Home values in America have increased since the 2008 slump in housing prices. Over the past six years, home prices have increased at an annual rate of 3.72%.

Which of the following functions models the increase in home values?

$A$  = accumulated value of the home over time

$P$  = the purchase price of the home

$t$  = time in years

$r$  = rate expressed as a decimal

- (A)  $A = P(0.0372)^t$
- (B)  $A = P(0.9628)^t$
- (C)  $A = P(1.0372)^t$

$$(D) A = P(3.72)^t$$

6. Home values in America have increased since the 2008 slump in housing prices. Over the past six years, home prices have increased at an annual rate of 3.72%. A home costing \$187,650 purchased in 2010 would be worth how much in 2013?

- (A) \$194,455
- (B) \$205,996
- (C) \$209,380
- (D) \$217,357

$$h(x) = bx^2 + 30$$

7. For the function  $h$  defined above,  $b$  is a constant and  $h(6) = 138$ . What is the value of  $h(-4)$ ?

- (A) 24
- (B) 64
- (C) 78
- (D) 86

8. Friendly Car Service charges a flat rate of \$1.50 and \$0.55 per mile. Mustafa has only \$11 to spend on his ride. Write an inequality that represents the maximum distance that Mustafa can travel.

- (A)  $0.55m - 1.50 \leq 11$
- (B)  $0.55m + 1.50 \leq 11$
- (C)  $1.50m + 0.55 \geq 11$
- (D)  $0.55m + 1.50 \geq 11$

$$S(x) = \frac{3}{5x} + 60$$

$$D(x) = 240 - x$$

9. The quantity of a product supplied to the public and the quantity of a product that is demanded by the public are each functions of the product's price. The functions above model the estimated supply and

demand of a particular product. The function  $S(x)$  gives the quantity of the product supplied to the market when the price is  $x$  dollars, and the function  $D(x)$  indicates the quantity of the product demanded when its price is  $x$  dollars.

What will be the effect on the quantity of the product supplied to the market if the price of the product is increased by \$15?

- (A) The quantity supplied will increase by 9 units.
- (B) The quantity supplied will decrease by 9 units.
- (C) The quantity supplied will increase by 36 units.
- (D) The quantity supplied will increase by 69 units.

$$S(x) = \frac{3}{5x} + 60$$
$$D(x) = 240 - x$$

10. The quantity of a product supplied to the public and the quantity of a product that is demanded by the public are each functions of the product's price. The functions above model the estimated supply and demand of a particular product. The function  $S(x)$  gives the quantity of the product supplied to the market when the price is  $x$  dollars, and the function  $D(x)$  indicates the quantity of the product demanded when its price is  $x$  dollars.

At what price will the supply of the product be equal to the demand for the product?

- (A) \$202.50
- (B) \$180.00
- (C) \$127.50
- (D) \$112.50

## Answer Explanations

1. **(D)** Substituting for  $m$  and  $n$  into  $3n - m$  results in the following.

$$3(4x^2 + 5x - 7) - (2x^3 + 3x - 11) =$$

$$12x^2 + 15x - 21 - 2x^3 - 3x + 11$$

Combine like terms by adding and subtracting those terms with variables raised to the same power.

$$-2x^3 + 12x^2 + 12x - 10$$

2. **(C)** The two functions  $f$  and  $g$  intersect when their  $x$ - and  $y$ -values are the same. Set the functions equal and solve for  $x$ .

$$6x^2 - 24 = -6x^2 + 24$$

Add  $6x^2$  and 24 to both sides to isolate  $x^2$ .

$$6x^2 - 24 + 6x^2 + 24 = -6x^2 + 24 + 6x^2 + 24$$

$$12x^2 = 48$$

$$x^2 = 4$$

Find the square root of each side of the equation.

$$x^2 = 4$$

$$x = \pm 2$$

Since we are looking for  $c$  rather than  $-c$ , its value is 2.

3. **(A)** The cost for  $m$  computers at \$18.75 each is  $\$18.75m$ . The cost for  $r$  keyboards at \$13.25 each is  $\$13.25r$ . The cost must be less than \$139.

$$18.75m + 13.25r < 139$$

4. **(D)** The cost of  $x$  gallons of water at \$0.17 per gallon is  $0.17x$ . The tax applied is found with  $0.17x + (0.073)(0.17x)$ , which can be rewritten as  $(1.073)(0.17x)$ . Add the monthly service charge to get

$$f(x) = 31.5 + (1.073)(0.17x)$$

5. **(C)**  $A$  = accumulated value of the home over time;  $P$  = the purchase price of the home;  $t$  = time in years. The formula for the growth of the home over time is the exponential equation  $A = P(1.0372)^t$ . The addition of 1 and 0.0372 ensures growth in the calculations.
6. **(C)** Input the information provided in the question into the formula  $A = P(1.0372)^t$ . We substitute 3 for  $t$  because three years have elapsed from 2010 to 2013.

$$A = 187,650(1.0372)^3 = \$209,380$$

7. **(C)** Replace  $h(x)$  with 138 and  $x$  with 6 to find  $b$ .

$$138 = b(6)^2 + 30$$

$$138 = 36b + 30$$

$$108 = 36b$$

$$3 = b$$

Replace  $b$  with 3 to find  $h(-4)$ .

$$h(-4) = 3(-4)^2 + 30$$

$$h(-4) = 78$$

8. **(B)** Let  $m$  equal the number of miles. Multiply  $m$  by the cost per mile, and add the fixed charge of \$1.50 per transaction.

$$0.55m + 1.50$$

This amount can be no more than \$11 since that is all the money that Mustafa has.

$$0.55m + 1.50 \leq 11$$

Choices (C) and (D) can be immediately eliminated as they express inequalities permitting costs no lower than \$11. Mustafa has only \$11 to spend.

9. **(A)** The quantity of the product supplied to the public is modeled by the function  $S(x) = \frac{3}{5}x + 60$ . If the price of the product is increased by \$15, the function becomes  $S(x + 15) = \frac{3}{5}(x + 15) + 60$ . To find the impact of increasing the price of the product by \$15, subtract  $S(x)$  from  $S(x + 15)$ .

$$\begin{aligned}S(x + 15) &= \frac{3}{5}x + 9 + 60 \\ &= \frac{3}{5}x + 69 \\ S(x) &= \frac{3}{5}x + 60 \\ S(x + 15) - S(x) &= \left(\frac{3}{5}x + 69\right) - \left(\frac{3}{5}x + 60\right) = 9\end{aligned}$$

An increase of \$15 in the price of the product yields an increase of 9 units of the product that is supplied to the public.

10. **(D)** To find the price of the product in which its demand equals supply, set the supply and demand functions equal to one another and solve for  $x$ .

$$\begin{aligned}S(x) &= \frac{3}{5}x + 60 \\ D(x) &= 240 - x \\ \frac{3}{5}x + 60 &= 240 - x \\ \frac{8}{5}x &= 180 \\ x &= \$112.50\end{aligned}$$

Choices (A), (B), and (C) yield prices in which the supply and demand functions are not equal.

## Drill 2

For multiple-choice questions, solve the problem and pick the correct answer from the provided choices. Each multiple-choice question has only one correct answer. For student-produced response questions, solve each problem and write down your answers separately.

1. The total area of a right circular cone can be expressed as  $TA = \pi rs + \pi r^2$  where  $TA$  is the total area,  $r$  is the radius of the base, and  $s$  is the slant height. Which of the following expresses  $s$  in terms of  $TA$  and  $r$ ?
  - (A)  $s = \frac{TA}{\pi r} - r$
  - (B)  $s = TA(\pi r - r)$
  - (C)  $s = (TA - r)\pi$
  - (D)  $s = \frac{TA}{\pi r - r}$
2. Gina is saving money for a cross-country road trip with two of her friends. She currently has \$240 saved for the excursion and can save \$125 each month in the future. Which of the following expressions represents the amount of money Gina will have saved after  $m$  months?
  - (A)  $365m$
  - (B)  $125 + 240m$
  - (C)  $240 + 125m$
  - (D)  $365m + 240$

$$2,500\left[1 + \frac{r}{1,200}\right]^{12}$$

3. The expression shown above provides the sum of money, in dollars, that is earned in a year by an initial deposit of \$2,500 in a savings institution that pays an annual rate of  $r\%$  compounded monthly. Which of the following expressions shows how much additional money is earned at an interest rate of 6% than at an interest rate of 3.2%?

- (A)  $2,500 \left[ 1 + \frac{6 - 3.2}{1,200} \right]^{12}$
- (B)  $2,500 \left[ 1 + \frac{\frac{6}{3.2}}{1,200} \right]^{12}$
- (C)  $\frac{2,500 \left[ 1 + \frac{6}{1,200} \right]^{12}}{2,500 \left[ 1 + \frac{3.2}{1,200} \right]^{12}}$
- (D)  $2,500 \left[ 1 + \frac{6}{1,200} \right]^{12} - 2,500 \left[ 1 + \frac{3.2}{1,200} \right]^{12}$

4. The standard form of an ellipse with major axis  $2a$  in the  $xy$ -plane is  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ . Which of the following expresses  $x$  in terms of  $y$ ,  $a$ , and  $b$ ?

- (A)  $x = \frac{(ab)^2}{y - x}$
- (B)  $x = \pm \sqrt{a^2 + b^2 - y^2}$
- (C)  $x = a^2 + b^2 - y^2$
- (D)  $x = \pm \frac{\sqrt{a^2 b^2 - a^2 y^2}}{b}$

$$P = -1.6x^2 + 2.6xy - 11.4$$

5. The equation above represents a profit function used by an event coordinator at a city museum. In the equation,  $P$  represents profit earned,  $x$  represents paid admissions ( $x > 0$ ), and  $y$  is any rental discount that is offered. Which of the following represents  $y$  in terms of  $x$  and  $P$ ?

- (A)  $y = \frac{1.6xP + 11.4}{2.6x}$
- (B)  $y = \frac{P + 1.6x^2 + 11.4}{2.6x}$
- (C)  $y = \frac{1.6x^2 + 11.4}{2.6x}$
- (D)  $y = \frac{P \pm \sqrt{2.6x}}{11.4}$

6. The density of an object is found by dividing the mass of the object by its volume. What is the volume, in milliliters, of an object with a mass of 48 grams and a density of 4 grams per milliliter?

- (A) 12
- (B) 14
- (C) 16
- (D) 24

7. A baseball is thrown into the air with an initial velocity of 64 feet per second. The formula  $h(t) = 64t - 16t^2$  provides the height,  $h$ , of the baseball after  $t$  seconds.

What will be the height of the baseball 2.3 seconds after it was thrown?

- (A) 71.43 feet
- (B) 62.56 feet
- (C) 53.69 feet
- (D) 41.37 feet

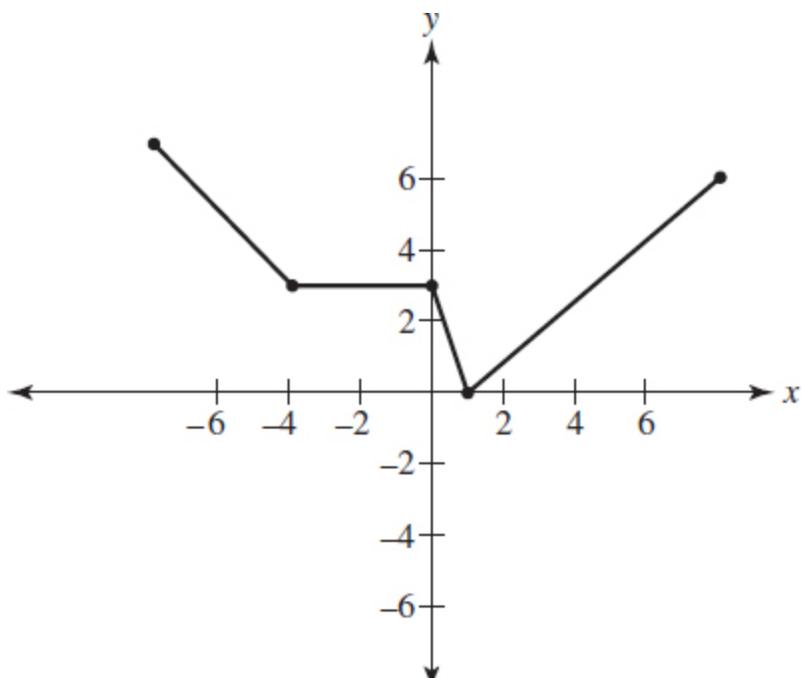
8. A baseball is thrown into the air with an initial velocity of 64 feet per second. The formula  $h(t) = 64t - 16t^2$  provides the height,  $h$ , of the baseball after  $t$  seconds.

To the nearest tenth of a second, how much time has elapsed when the ball is 41 feet above the ground?

- (A) 3.2
- (B) 2.1 and 3.6
- (C) 0.8
- (D) 0.8 and 3.2

$$(4,264 + 200m^2) + 6(12m^2 - 125)$$

9. The expression shown above can be written in the form of  $am^2 + b$ , where  $a$  and  $b$  are constants. What is the value of  $2a + b$ ?



10. The complete graph of the function  $g$  is shown in the  $xy$ -plane above. What value of  $x$  yields the minimum value of  $g(x)$ ?
- (A) 0  
 (B) 1  
 (C) 2  
 (D) 5

### Answer Explanations

1. **(A)** Begin isolating  $s$  by first factoring the greatest common factor.

$$TA = \pi rs + \pi r^2$$

$$TA = \pi r(s + r)$$

Divide both sides of the equation by  $\pi r$ .

$$\frac{TA}{\pi r} = \frac{\pi r(s + r)}{\pi r}$$

$$\frac{TA}{\pi r} = s + r$$

Subtract  $r$  from each side of the equation.

$$\begin{aligned}\frac{TA}{\pi r} - r &= s + r - r \\ \frac{TA}{\pi r} - r &= s\end{aligned}$$

2. **(C)** Gina has \$240 saved, so that sum will be part of the answer. Each month, for  $m$  months, she will save \$125. Therefore, she will save an additional  $\$125m$  after  $m$  months. By adding the initial savings and the monthly accumulation of money, we arrive at  $240 + 125m$  as the amount saved after  $m$  months.
3. **(D)** The formula indicates the amount of money earned at a savings institution on a \$2,500 deposit, compounded monthly at  $r\%$ . To find the additional dollars earned at 6% versus 3.2%, find the difference between the annual earnings at each rate.

$2,500\left[1 + \frac{6}{1,200}\right]^{12}$  represents the amount earned each year at 6%.

$2,500\left[1 + \frac{3.2}{1,200}\right]^{12}$  represents the amount earned at 3.2%.

Find their difference by subtracting the smaller expression from the larger.

$$2,500\left[1 + \frac{6}{1,200}\right]^{12} - 2,500\left[1 + \frac{3.2}{1,200}\right]^{12}$$

4. **(D)** Begin isolating  $x$  by multiplying the equation by  $a^2b^2$ .

$$\begin{aligned}
a^2b^2\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1\right) \\
b^2x^2 + a^2y^2 &= a^2b^2 \\
b^2x^2 &= a^2b^2 - a^2y^2 \\
\frac{b^2x^2}{b^2} &= \frac{a^2b^2 - a^2y^2}{b^2} \\
x^2 &= \frac{a^2b^2 - a^2y^2}{b^2} \\
\sqrt{x^2} &= \pm\sqrt{\frac{a^2b^2 - a^2y^2}{b^2}} \\
x &= \pm\frac{\sqrt{a^2b^2 - a^2y^2}}{b^2}
\end{aligned}$$

5. **(B)** Use the basic steps of algebra to isolate  $y$ .

**STEP 1** Add  $1.6x^2$  and  $11.4$  to both sides of the equation.

$$\begin{aligned}
P &= -1.6x^2 + 2.6xy - 11.4 \\
P + 1.6x^2 + 11.4 &= -1.6x^2 + 2.6xy - 11.4 + 1.6x^2 + 11.4 \\
P + 1.6x^2 + 11.4 &= 2.6xy
\end{aligned}$$

**STEP 2** Divide both sides of the equation by  $2.6x$ .

$$\begin{aligned}
\frac{P + 1.6x^2 + 11.4}{2.6x} &= \frac{2.6xy}{2.6x} \\
\frac{P + 1.6x^2 + 11.4}{2.6x} &= y
\end{aligned}$$

6. **(A)** The density of an object is equal to the mass of the object divided by the volume of the object, which can be expressed as density = mass/volume. Thus, if an object has a density of 4 grams per milliliter and a mass of 48 grams, the equation becomes the following.

$$\begin{aligned}
4 &= \frac{48}{V} \\
4V &= 48 \\
V &= 12
\end{aligned}$$

The volume, then, is 12 milliliters/gram.

7. **(B)** Find  $h(2.3)$ .

$$h(2.3) = (64)(2.3) - (16)(2.3^2) = 62.56 \text{ feet}$$

8. **(D)** Replace  $h(t)$  with 41.

$$41 = 64t - 16t^2$$

Set the equation equal to 0 and solve.

$$16t^2 - 64t + 41 = 0$$

$$a = 16, b = -64, c = 41$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-(-64) \pm \sqrt{(-64)^2 - 4(16)(41)}}{2(16)}$$

$$t = 0.8 \text{ and } t = 3.2$$

9. **4,058** Simplify  $6(12m^2 - 125)$  and combine like terms.

$$(4,264 + 200m^2) + 6(12m^2 - 125)$$

$$4,264 + 200m^2 + 72m^2 - 750$$

$$272m^2 + 3,514$$

This calculation shows that  $a = 272$  and  $b = 3,514$ . Therefore,  $2a + b$  equals  $2(272) + 3,514 = 4,058$ .

10. **(B)** The minimum value of  $g(x)$  occurs at  $(1, 0)$  on the graph. The value of  $x$ , then, that yields the minimum value of  $g(x)$  is 1.

## Problem Solving and Data Analysis

The Problem Solving and Data Analysis portion of the digital SAT Math section will require the test taker to be proficient in the following skills:

### TIP

Many of the questions depict surveys conducted in certain environments. The test taker must be certain that survey respondents will be randomly selected and the surveys contain at least 50 respondents. In addition, be careful of biased communities. For example, if a city government surveyed people about the need for additional funds for dog parks, survey all groups. A survey conducted among a group of dog owners would likely be biased.

- Ratios, rates, and proportional relationships
- Percentages
- One-variable data: distributions and measures of center and spread
- Two-variable data models and scatter plots
- Probability and conditional probability
- Inference from sample statistics and margin of error
- Evaluating statistical claims: observational studies and experiments

### Measures of Center Drill

For multiple-choice questions, solve the problem and pick the correct answer from the provided choices. Each multiple-choice question has only one correct answer. For student-produced response questions, solve each problem and write down your answers separately.

1. There are nine students in a statistics class. After a recent test, the average (arithmetic mean) score was 79%. What must a tenth

student's score be to raise the class average to 81%?

- (A) 87%
  - (B) 89%
  - (C) 94%
  - (D) 99%
2. The average (arithmetic mean) of  $2x - 4$ ,  $x + 4$ , and  $3x - 6$  is  $x + 4$ . What is the value of  $-2x + 3$ ?
- (A)  $-9$
  - (B)  $-6$
  - (C)  $6$
  - (D)  $12$
3. At Santa Fe Elementary School, the 59 students in 4th grade participated in a canned food drive. The table below shows the number of cans contributed by each student.

Number of Cans per Student	Frequency
8	14
9	16
10	7
11	7
12	8
13	5
14	2

Which number of cans contains the median of this data set?

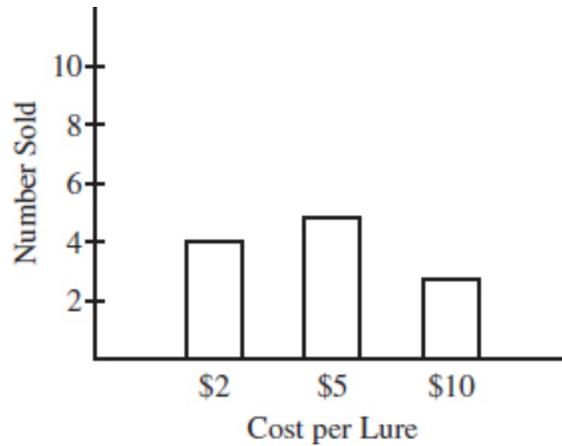
- (A) 11
- (B) 10
- (C) 9
- (D) 8

4. A world history class recently took its midterm exam. The following table represents the array of scores.

		<b>x</b>	<b>x</b>	
		<b>x</b>	<b>x</b>	
<b>x</b>		<b>x</b>	<b>x</b>	
<b>x</b>	<b>x</b>	<b>x</b>	<b>x</b>	<b>x</b>
<b>20%</b>	<b>40%</b>	<b>60%</b>	<b>80%</b>	<b>100%</b>

If a thirteenth student took the exam the next day, what must that score be to arrive at a class mean score of 60%?

5. Seven students draw numbered cards, each with a value that is a positive integer. The mean of their selections is 14. What is the largest number any student could have drawn?
- (A) 92  
(B) 70  
(C) 28  
(D) 14
6. Seven people each select an index card. Every card has a positive integer printed on one side. If the mean value of the cards is 15, what is the greatest value any one card can have?
- (A) 69  
(B) 79  
(C) 89  
(D) 99



7. The bar graph above represents the fishing lures sold in a sales competition sponsored by the seller. What is the median cost of the lures sold?
8. If  $x$  is the average (arithmetic mean) of  $2p$  and  $12$ ,  $y$  is the average of  $5p$  and  $18$ , and  $z$  is the average of  $8p - 6$ , what is the average of  $x$ ,  $y$ , and  $z$  in terms of  $p$ ?
- (A)  $5p + 8$   
 (B)  $3p + 6$   
 (C)  $p + 12$   
 (D)  $p + 6$

**Percent of Homeowners Using Solar Power Fort Myers, Florida**

District	Percent of Homeowners Using Solar Power
1	28.1
2	13.6
3	38.3
4	17.9
5	11.6

6	17.4
7	22.0

9. An analysis of all homeowners in Fort Myers, Florida, was conducted to determine the percentage of those who use solar power for their electrical needs. The city has 27 districts, of which 7 are listed in the table above. The median percent of all homeowners who use solar power in Fort Myers is 16.8%. What is the difference between the median of the 7 districts listed above and the median percent of the entire city?
- (A) 26.7%  
(B) 4.5%  
(C) 2.3%  
(D) 1.1%
10. If  $M$  represents the number of prime numbers less than 25 and  $r$  is their range, what is  $2M - r$ ?
- (A)  $-3$   
(B)  $3$   
(C)  $8.9$   
(D)  $27$

### Answer Explanations

1. **(D)** If nine students earned an average score of 79%, the class amassed 711 points because  $9 \times 79 = 711$ . In order for ten students to have an average score of 81, the class must collect 810 points because  $10 \times 81 = 810$ . By subtracting we get  $810 - 711 = 99$ . The tenth student must earn a score of 99% to raise the class average to 81%.
2. **(A)** Add the three terms and divide the sum by 3. Set this value equal to  $x + 4$ , which is the average.

$$\frac{2x - 4 + x + 4 + 3x - 6}{3} = x + 4$$

$$\frac{6x - 6}{3} = x + 4$$

$$6x - 6 = 3x + 12$$

$$3x = 18$$

$$x = 6$$

Replace  $x$  with 6 in  $-2x + 3$ .

$$-2(6) + 3 = -9$$

3. **(C)** You are told that 59 students participated in the canned food drive. The table arranges the number of cans per student in ascending order. Thus, the median student contribution is the number of cans contributed by the 30th student. Add the frequency of contributions until you arrive at the 30th student.

Number of Cans per Student	Frequency
8	14
9	16

Adding 14 and 16 yields a total of 30. The 30th student lies in the category of students who contributed 9 cans each.

4. **40** Add the scores of all 12 of the students.

$$(2 \times 20) + (1 \times 40) + (4 \times 60) + (4 \times 80) + (1 \times 100) = 740$$

In order for 13 students to average 60%, the class must amass 780 points.

$$13 \times 60 = 780$$

Since the first 12 students accumulated 740 points, the thirteenth student must score a 40% on the test.

$$780 - 740 = 40$$

5. **(A)** Find the sum of the cards by multiplying the mean by the number of cards.

$$7 \times 14 = 98$$

To maximize the value of a single card, minimize the value of the other cards by assigning each a value of 1:

$$1 + 1 + 1 + 1 + 1 + 1 + 92 = 98$$

6. **(D)** Since 7 cards have a mean value of 15, their sum is 105.

$$7 \times 15 = 105$$

If 6 of the cards have the number 1 on them, then 1 card would have 99.

$$105 - (6 \times 1) = 99$$

7. **5** The median of a data set is the middle value when the data are arranged in order. Array the number of lures sold; there are four \$2 lures, five \$5 lures, and three \$10 lures.

$$2 \ 2 \ 2 \ 2 \ 5 \ 5 \ 5 \ 5 \ 5 \ 10 \ 10 \ 10$$

The middle numbers are both 5, so find their mean.

$$(5 + 5) \div 2 = 5$$

8. **(A)** Find the average of  $x$ ,  $y$ , and  $z$  by calculating their sum and dividing by 3.

$$\frac{(2p + 12) + (5p + 18) + (8p - 6)}{3} = \frac{15p + 24}{3} = 5p + 8$$

9. **(D)** Arrange the percentages in the table in ascending order.

11.6 13.6 17.4 17.9 22.0 28.1 38.3

The median of the data shown is 17.9% because it is the middle term. Subtract 16.8%, the median of all homeowners in Fort Myers, from 17.9%.

$$17.9\% - 16.8\% = 1.1\%$$

10. **(A)** The prime numbers less than 25 are 2, 3, 5, 7, 11, 13, 17, 19, 23. There are nine prime numbers less than 25, so  $M = 9$ . Find the range of this set of numbers by subtracting the lowest value from the highest.

$$23 - 2 = 21$$

The range of the numbers is 21, so  $r = 21$ .

$$2M - r = 2(9) - 21 = -3$$

## Unit Conversion Drill

For multiple-choice questions, solve the problem and pick the correct answer from the provided choices. Each multiple-choice question has only one correct answer. For student-produced response questions, solve each problem and write down your answers separately.

1. Jenna needs to take a taxicab ride in Chicago to go to an important job interview. The taxi service she has selected provides the following fee schedule.

First  $\frac{1}{4}$  mile: \$2.00

Each  $\frac{1}{4}$  mile after: \$1.25

Excluding a tip, Jenna's fare was \$15.75. How many miles did she travel?

- (A) 2
  - (B) 2.5
  - (C) 2.75
  - (D) 3
2. Gold has always been coveted for its aesthetic qualities. However, due to its malleability, it also has many industrial and electronic applications. Gold can be stretched so thinly that a single ounce can cover 100 square yards of surface. How many square feet can be covered with 2.5 ounces of gold?
    - (A) 2,250 square feet
    - (B) 1,125 square feet
    - (C) 750 square feet
    - (D) 150 square feet
  3. The legend on a map shows that 1 inch = 80 miles. The distance on a map shows that Hartford, Connecticut, and Roanoke, Virginia, are 5.05 inches apart. What is the actual distance between the two cities?
    - (A) 80 miles
    - (B) 205 miles
    - (C) 395 miles
    - (D) 404 miles
  4. Homeowners want to create a circular rock garden that measures 20 feet in diameter. Reference materials state that the space for the garden needs to be 3" deep. The rocks they wish to buy cost \$80 per cubic

yard and must be purchased in whole cubic yards. How much will they have to spend on rocks?

- (A) \$480
- (B) \$400
- (C) \$320
- (D) \$240

<b>BMI</b>	
Underweight	Below 18.5
Normal	18.5–24.9
Overweight	25.0–29.9
Obese	30.0 and above

5. Body mass index (BMI) is a calculation that measures obesity and the potential for serious health risks such as strokes and heart attacks. The BMI can be found by using the following formula.

$$\frac{\text{weight (pounds)}}{\text{height}^2 \text{ (inches)}} \times 703$$

What is the category of BMI of a patient who weighs 182 pounds and is 5'10"?

- (A) Underweight
- (B) Normal
- (C) Overweight
- (D) Obese

<b>BMI</b>	
Underweight	Below 18.5
Normal	18.5–24.9
Overweight	25.0–29.9
Obese	30.0 and above

6. Body mass index is a calculation that measures obesity and the potential for serious health risks such as strokes and heart attacks. The BMI can be found by using the following formula.

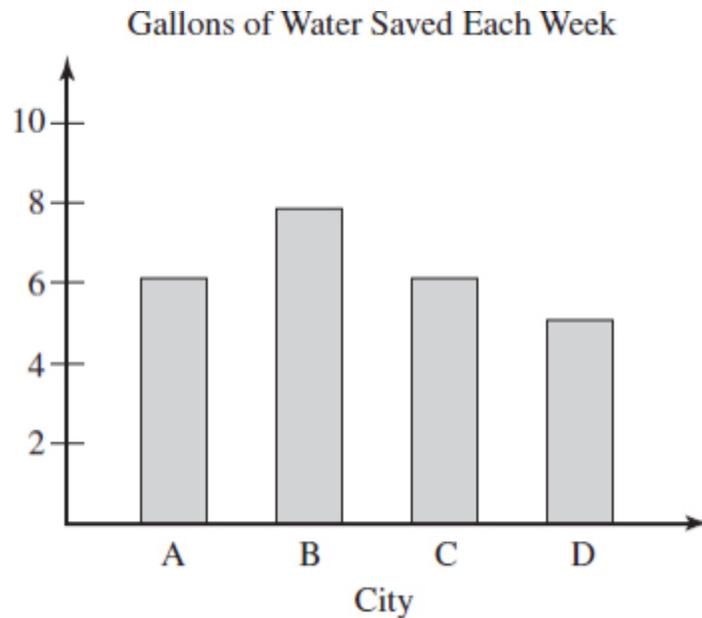
$$\frac{\text{weight (pounds)}}{\text{height}^2 \text{ (inches)}} \times 703$$

What is the weight of a patient who is 6 feet tall and has the lowest BMI that is categorized as obese? (Round to the nearest pound.)

- (A) 194
- (B) 221
- (C) 231
- (D) 241

1 hectogram = 100 grams  
100 centigrams = 1 gram

7. A popular snack is packaged in 2-hectogram containers. If a customer ate  $\frac{1}{4}$  of one of these containers, how many centigrams did the customer consume?
- (A) 50
  - (B) 100
  - (C) 1,000
  - (D) 5,000



8. Certain cities in the southwestern United States are experimenting with water-saving plumbing. If the total gallons saved as expressed in the graph above equal 25,000 gallons, what is an appropriate label for the vertical axis?
- (A) Gallons saved (in tens)  
 (B) Gallons saved (in hundreds)  
 (C) Gallons saved (in thousands)  
 (D) Gallons saved (in hundred thousands)
9. On July 15, 2015, the *New Horizons* planet orbiter transmitted its first pictures of minor planet Pluto. Pluto is approximately 3 billion miles distant from Earth. If the transmission of the photographs traveled at the speed of light, to the nearest hundredth, how many hours did it take for the photographs to reach Earth from Pluto? (Note: light travels 186,000 miles per second)
10. The specific gravity of a substance is the ratio of the weight of the substance compared weight to the volume of an equal weight of water. If water weighs 62.5 pounds per cubic foot, what is the weight in pounds per cubic foot of a liquid that has a specific gravity of 3?
- (A) 0.047

- (B) 130
- (C) 187.5
- (D) 260

### Answer Explanations

1. **(D)** Subtract \$2.00 from \$15.75 to count the first quarter mile.

$$\$15.75 - \$2.00 = \$13.75$$

Divide \$13.75 by \$1.25 to calculate the remaining quarter miles.

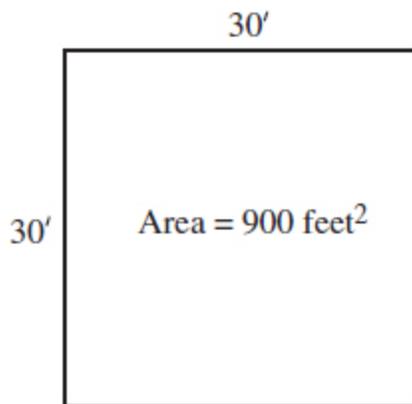
$$\$13.75 \div \$1.25 = 11$$

The taxi traveled a total of 12 quarter miles. Divide 12 by 4 to find the equivalent number of miles.

$$12 \div 4 = 3$$

2. **(A)** An ounce of gold covers 100 square yards of surface, but the question requires the answer to be in square feet.

$$1 \text{ yard} = 3 \text{ feet}$$



100 square yards is equivalent to 900 square feet, so one ounce of gold covers 900 square feet. Multiply 900 by 2.5 to find the area covered by 2.5 ounces of gold.

$$2.5 \times 900 = 2,250 \text{ square feet}$$

3. **(D)** Use the proportion  $\frac{\text{map distance}}{\text{actual distance}} = \frac{\text{map distance}}{\text{actual distance}}$  to answer the question.

$$\begin{aligned}\frac{1}{80} &= \frac{5.05}{d} \\ 1 \times d &= 80 \times 5.05 \\ d &= 404\end{aligned}$$

The distance between Hartford and Roanoke is 404 miles. Choice (A) can be immediately eliminated; 1 inch = 80 miles and the map shows 5.05 inches.

4. **(D)** Convert everything into yards.

$$\begin{aligned}3 \text{ inches} &= 0.08 \text{ yards} \\ 10\text{-foot radius} &= 3.33 \text{ yards} \\ \pi &= 3.14\end{aligned}$$

Use the formula  $V = \pi r^2 h$ , the volume of a cylinder, to determine how much rock is needed if 3.33 yards is the radius and 0.08 yards is the height.

$$3.14 \times 3.33^2 \times 0.08 = 3.14 \times 11.09 \times 0.08 = 2.79 \text{ cubic yards}$$

Rounding to the nearest cubic yard, the homeowners must purchase 3 cubic yards of rock.

$$3 \text{ cubic yards} \times \$80 = \$240$$

5. **(C)** Use the BMI formula with the provided data to assess the patient's BMI number. Remember to convert the height to inches: 5 feet 10 inches equals 70 inches.

$$\frac{182}{70^2} \times 703 = 26.11$$

A patient with a BMI of 26.11 is classified as overweight.

6. **(B)** Use the BMI formula with the provided data to assess the patient's BMI weight. Remember to convert the height to inches: 6 feet equals 72 inches. The lowest BMI categorized as obese is 30.0. Let  $x$  = the patient's weight.

$$\begin{aligned}\frac{x}{72^2} \times 703 &= 30 \\ 0.136x &= 30 \\ x &\approx 221\end{aligned}$$

7. **(D)** One hectogram equals 100 grams, so 2 hectograms equal 200 grams. The customer consumed  $\frac{1}{4}$  of one container, so 50 grams of the snack was consumed.

$$\frac{1}{4} \times 200 = 50$$

Each gram equals 100 centigrams, so multiply 50 by 100 to find the amount consumed expressed as centigrams.

$$50 \times 100 = 5,000$$

8. **(C)** The sum of the measures of the bars in the graph is 25 units. Given that the four cities saved 25,000 gallons of water, the vertical axis should read *Gallons saved (in thousands)* because  $25,000 \div 25 = 1,000$ .
9. **4.48** Find the quotient of 3 billion and 186,000 to find the number of seconds elapsed between the transmission and reception of the photos (these are known as "light-seconds").

$$3,000,000,000 \div 186,000 = 16,129 \text{ seconds}$$

There are 60 seconds in a minute and 60 minutes in an hour, so there are 3,600 seconds in one hour ( $60 \times 60 = 3,600$ ). Divide 16,129 by 3,600 to find the number of hours needed to successfully transmit photographs from Pluto to Earth.

$$16,129 \div 3,600 = 4.48 \text{ hours}$$

10. (C) Use the ratio  $\frac{62.5 \text{ pounds}}{1 \text{ cubic foot}}$  to find the weight of a liquid that has a specific gravity of 3.

$$\begin{aligned}\frac{62.5}{1} &= \frac{x}{3} \\ (3)(62.5) &= x \\ 187.5 &= x\end{aligned}$$

## Percentages

### Drill 1

For multiple-choice questions, solve the problem and pick the correct answer from the provided choices. Each multiple-choice question has only one correct answer. For student-produced response questions, solve each problem and write down your answers separately.

1. The Environmental Protection Agency (EPA) has demonstrated that newer automobile emission standards have had a strong positive correlation with cleaner air in large metropolitan areas. Thus, it is the EPA's responsibility to get older cars off the road.

### Cars in Operation in 1997

Age of Car

Millions of

	Cars
<3 years old	21.5
3–5 years old	29.9
6–8 years old	22.2
9–11 years old	16.7
≥12 years old	14.4

What percent of cars in operation in 1997 were between 6 and 11 years old?

- (A) 37.2
- (B) 41.3
- (C) 48.1
- (D) 53.2

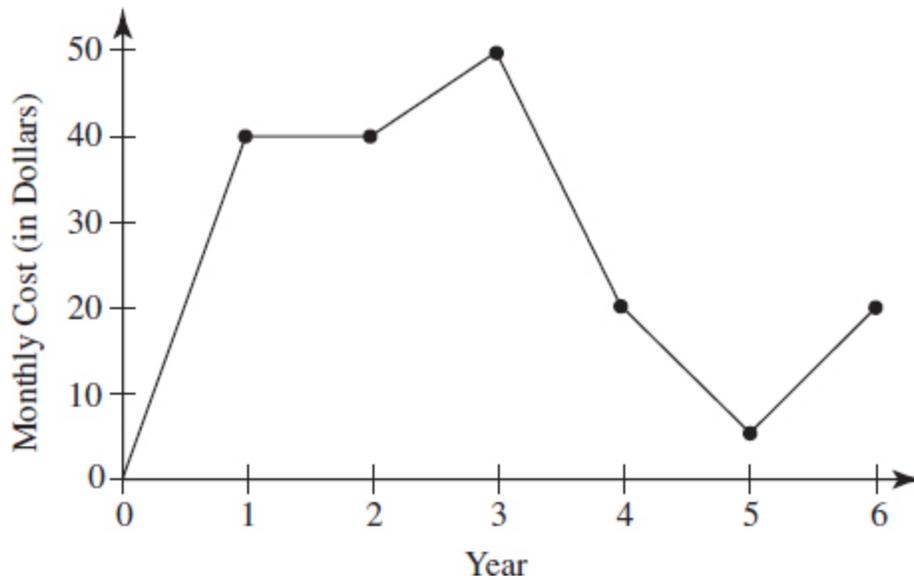
2. A retailer was unable to sell all of her sofas, so she reduced the price of a sofa by 40%. Once she saw the sofas were beginning to sell, she increased the price by 40%. After the 40% price increase, what percent is the new price of the original price?

- (A) 100%
- (B) 84%
- (C) 75%
- (D) 60%

Rank	Country	GDP-per capita (US\$)
1	Liechtenstein	141,100
2	Qatar	104,300
3	Luxembourg	81,100
4	Bermuda	69,900
5	Monaco	63,400
6	Singapore	60,500
7	Jersey	57,000

8	Falkland Islands (Islas Malvinas)	55,400
9	Norway	54,200
10	Brunei	50,000

3. The gross domestic product (GDP) is a measure of a country's wealth. The per capita income for a country is found by dividing the sum of all the goods and services that a country produces by the country's population. Which country exceeded the per capita income of Singapore by approximately 4.8%?
- (A) Qatar
  - (B) Monaco
  - (C) Bermuda
  - (D) Norway
4. A customer service call center for a credit card company asks 1 in 8 of the callers to take a survey at the conclusion of the call. Approximately 20% of those solicited consent to the survey. If 400 callers were serviced between 2:00 and 3:00 one afternoon and 20% of the callers consented to the survey, how many callers did not consent to the survey?
- (A) 20
  - (B) 30
  - (C) 35
  - (A) 40



5. The monthly costs of certain internet plans have varied greatly over the six-year period as illustrated in the graph. What was the largest percent drop in price from one year to another?
- (A) 25  
 (B) 50  
 (C) 75  
 (D) 85  
 (E)
6. A recent marathon featured runners of many different ages. The table below shows a listing of the participants as categorized by age and place in the race.

Age	Under 20	20–29	30–39	40–49	50–69	70 and Over	Total
Top 10 finisher	2	6	1	1	0	0	10
Finished in places 11 to 40	11	11	5	3	0	0	30
Finished after 40th place	5	8	23	31	43	12	122
Total	18	25	29	35	43	12	162

What percent of the top forty finishers came from the 30 to 39 age group?

- (A) 8%
- (B) 10%
- (C) 15%
- (D) 18%

7. A recent marathon featured runners of many different ages. The table below shows a listing of the participants as categorized by age and place in the race.

Age	Under 20	20–29	30–39	40–49	50–69	70 and Over	Total
Top 10 finisher	2	6	1	1	0	0	10
Finished in places 11 to 40	11	11	5	3	0	0	30
Finished after 40th place	5	8	23	31	43	12	122
Total	18	25	29	35	43	12	162

What is the difference between the percent of runners under 20 years of age who finished in places 11 to 40 and the percent of runners aged 40 to 49 who also finished in places 11 to 40?

- (A) 52.5%
- (B) 46.4%
- (C) 34.2%
- (D) 31.1%

#### AP Tests Taken

Gender	U.S. History	Literature	French	Total
Male	42	31	12	85
Female	33	42	26	101
Total	75	75	38	186

8. The table above shows the number of 11th-grade students at a local high school who took the following AP tests: U.S. History, Literature, and French. Which category accounts for approximately 17.7% of the test takers?
- (A) Males taking the U.S. History test  
 (B) Males taking the French test  
 (C) Females taking the Literature test  
 (D) Females taking the U.S. History test
9. Briana bought a pair of running shoes at a 25% discount off the original price. The total she paid the cashier was  $r$  dollars, which includes an 8% sales tax on the discounted price. Which of the following represents the original price of the running shoes in terms of  $r$ ?
- (A)  $0.81r$   
 (B)  $\frac{r}{0.75}$   
 (C)  $(0.75)(1.08)r$   
 (D)  $\frac{r}{(1.08)(0.75)}$
10. A pair of hiking boots sells for \$80.00. After one week, the vendor reduced the cost of the boots by 20%. After two weeks, he raised the current price by 20%. What is the cost of the hiking boots after two weeks? (Ignore the \$ sign.)

### Answer Explanations

1. **(A)** Use the formula  $\frac{\text{part}}{\text{whole}} = \frac{n}{100}$  to find the percent of those operating cars that were 6 to 11 years old.

6–8: 22.2

9–11: 16.7

The number of cars (in millions) that are between 6 and 11 years old is 38.9.

$$22.2 + 16.7 = 38.9$$

Total cars (in millions): 104.7

$$\frac{3.89}{104.7} = \frac{n}{100}$$

Cross-multiply and solve for  $n$ .

$$\begin{aligned} 104.7n &= 38,900 \\ n &= 37.2\% \end{aligned}$$

2. **(B)** To facilitate making calculations, assume the original price was \$100. Find the price after the reduction.

$$100 - (0.40)(100) = 60$$

Next, find the price after the increase.

$$60 + (0.40)(60) = 84$$

The new price is  $\frac{84}{100}$  of the original price, which is 84%.

3. **(B)** The per-capita income of Singapore is \$60,500. Multiply that value by 0.048 and add it to \$60,500.

$$\$60,500 + (0.048)(\$60,500) = \$63,404$$

At \$63,400, Monaco's per capita income is closest to this value.

4. **(D)** If 1 in 8 callers were solicited for the survey, then one-eighth of the callers were asked to participate. Thus, one-eighth of 400 callers is 50 callers. Since 20% of the callers consented to the survey, then 80% did not want to take part. Find 80% of 50 by multiplying 50 by 0.8.

$$50 \times 0.8 = 40$$

5. **(C)** The greatest drop, as a percentage, occurred between years 4 and 5.

Year 4: \$20

Year 5: \$5

Use the formula  $\frac{\text{decrease}}{\text{original}} = \frac{n}{100}$  to find the percent decrease.

$$\begin{aligned}\frac{15}{20} &= \frac{n}{100} \\ 20n &= 1,500 \\ n &= 75\%\end{aligned}$$

6. **(C)** Of the runners who placed in the top 40, 6 were in the age group of 30 to 39. One runner in that age group finished in the top 10, and 5 others finished somewhere in places between 11 and 40. Divide 6 by 40 to get 0.15, which, when expressed as a percent, is 15%.
7. **(A)** The number of runners under 20 years of age who placed in positions 11 through 40 was 11. Given that the total number of runners in that age group was 18, the percent who finished in places 11 through 40 is 61.1%. The number of runners between the ages of 40 and 49 who also finished in places 11 through 40 was 3. Given that a total of 35 runners were in this category, the percent of those runners who finished in places 11 through 40 was 8.6%. The difference between 61.1% and 8.6% is 52.5%.
8. **(D)** Find 17.7% of 186, which is the total number of students who took the AP tests.

$$(0.177)(186) = 32.92$$

9. **(D)** Let  $x$  be the original price of the running shoes, in dollars. The discounted price is 25 percent off the original price, so  $x - 0.25x = 0.75x$  is the discounted price, in dollars. The tax is 8 percent of the discounted price, so  $0.08(0.75x)$  is the tax on the purchase, in dollars.

The price  $r$ , in dollars, that Briana paid the cashier is the sum of the discounted price and the tax.

$$r = 0.75x + (0.08)(0.75x)$$

This can be rewritten as  $r = 1.08(0.75x)$ . Therefore, the original price,  $x$ , of the running shoes, in dollars, can be written as  $\frac{r}{(1.08)(0.75)}$ .

10. **76.8** Find the cost of the boots after one week.

$$\$80.00 - (0.20)(\$80.00) = \$64.00$$

Add 20% to \$64.00 to find the cost after two weeks.

$$\$64.00 + (0.20)(\$64.00) = \$76.80$$

## Drill 2

For multiple-choice questions, solve the problem and pick the correct answer from the provided choices. Each multiple-choice question has only one correct answer. For student-produced response questions, solve each problem and write down your answers separately.

1. What number is 175% of 144?
  - (A) 252
  - (B) 192
  - (C) 108
  - (D) 82.3
2. A high school's 10th, 11th, and 12th grades represent 78% of the student body. What fraction of the student body is the freshman class?

- (A)  $\frac{39}{50}$
- (B)  $\frac{67}{100}$
- (C)  $\frac{11}{50}$
- (D)  $\frac{1}{50}$

3. Sean owed  $p$  dollars for a loan he secured for college expenses. He promised to pay off the debt in monthly installments, with each installment equal to 20% of the existing debt. After two monthly payments, which expression reflects the balance due on his loan?

- (A)  $0.8p$
- (B)  $0.64p$
- (C)  $0.6p$
- (D)  $0.48p$

4. Connor surveyed a random sample of 80 students in his class regarding the need for additional crosswalks at the school. Only 15.6% of the students felt they were necessary. Of the 320 students in his sophomore class, about how many would favor additional crosswalks?

- (A) 50
- (B) 58
- (C) 71
- (D) 74

Year	Hits
2021	2,150
2022	3,230

5. Steve's website statistics page shows the number of hits the website received in 2021 and 2022 as shown in the table above. He believes the percentage increase from 2021 to 2022 will be triple the percentage increase from 2022 to 2023. If Steve is correct, which of the following is closest to the website's projected hits in 2023?

- (A) 1,925

- (B) 2,150
- (C) 3,770
- (D) 4,430

6. Roasted asparagus (134 grams) provides the percentages of minimum daily nutritional requirements shown in the table below.

Nutrient	Percentage of Daily Requirement
Vitamin A	20
Calcium	3
Vitamin D	0
Vitamin B <sub>12</sub>	0
Vitamin C	12
Iron	16
Vitamin B <sub>6</sub>	5
Magnesium	5
Protein	9

A red bell pepper (92 grams) provides approximately 123% of the daily minimum requirement of vitamin C. Which of the following is closest to the number of grams of roasted asparagus that is equivalent to this amount?

- (A) 1,600
- (B) 1,400
- (C) 1,200
- (D) 1,000

7. Roasted asparagus (134 grams) provides the percentages of minimum daily nutritional requirements shown in the table below.

Nutrient	Percentage of Daily Requirement
----------	---------------------------------

	Requirement
Vitamin A	20
Calcium	3
Vitamin D	0
Vitamin B <sub>12</sub>	0
Vitamin C	12
Iron	16
Vitamin B <sub>6</sub>	5
Magnesium	5
Protein	9

Which of the following is closest to the number of grams of roasted asparagus needed to fulfill a person's minimum daily requirement of iron?

- (A) 800
- (B) 1,000
- (C) 1,200
- (D) 1,600

8. Nutritionists recommend the daily consumption of 4,700 milligrams of potassium. Men generally consume 3,200 milligrams per day, and women consume about 2,300 milligrams per day. The table below shows a list of potassium-rich foods as recommended by nutrition experts.

Foods with Potassium	Serving Size	Potassium (mg)
Apricots, dried	10 halves	407
Avocados, raw	1 ounce	180
Bananas, raw	1 cup	594
Beets, cooked	1 cup	519
Brussel	1 cup	504

sprouts, cooked		
Cantaloupe	1 cup	494
Dates, dry	5 dates	271
Figs, dry	2 figs	271
Kiwifruit, raw	1 medium	252
Lima beans	1 cup	955
Melons, honeydew	1 cup	461
Milk, fat free or skim	1 cup	407
Nectarines	1 nectarine	288
Orange juice	1 cup	496
Oranges	1 orange	237
Pears (fresh)	1 pear	208
Peanuts, dry roasted, unsalted	1 ounce	187
Potatoes, baked	1 potato	1,081
Prune juice	1 cup	707
Prunes, dried	1 cup	828
Raisins	1 cup	1,089
Spinach, cooked	1 cup	839
Tomato products, canned sauce	1 cup	909
Winter squash	1 cup	896
Yogurt, plain, skim milk	8 ounces	579

Jake read that the headaches he was enduring after running were caused by potassium depletion. Although he was consuming 3,200 milligrams of potassium daily, he was told he needed at least 10% more potassium than was recommended by nutritionists. To accrue

this amount, Jake has decided to consume an extra potato and a cup of lima beans in his daily diet. If he consumes these extra items each day, how much more potassium will he consume than is recommended for his augmented need of this nutrient?

9. A supermarket chain received an average grade (arithmetic mean) of 65% customer satisfaction from 10 recent customer surveys. The chain manager wants to earn at least an 80% rating from its first 20 customer surveys. What is the lowest score the 17th survey can receive and still achieve the 80% satisfaction rate from its first 20 surveys? (Omit the percent sign.)
10. The sum of three numbers is 780. One of the numbers,  $n$ , is 50% more than the sum of the other two numbers. What is the value of  $n$ ?
- (A) 144  
(B) 312  
(C) 424  
(D) 468

### Answer Explanations

1. **(A)** 175% of a number is greater than 100% of a number. Thus, eliminate choices (C) and (D) as they are less than 144. Let  $x$  represent the unknown number. Convert 175% to a calculator friendly 1.75.

$$\begin{aligned}(1.75)(144) &= x \\ 252 &= x\end{aligned}$$

2. **(C)** Begin by finding the percent of the class represented by the freshman class.

$$100\% - 78\% = 22\%$$

Convert 22% into a fraction and simplify.

$$22\% = \frac{22}{100} = \frac{11}{50}$$

3. **(B)** After the first month of paying off 20% of his student loan, Sean's remaining balance looks like this.

$$p - 0.2p = 0.8p$$

Sean's loan balance after one month was  $0.8p$ . He now must pay off 20% of the  $0.8p$  that remains as the loan balance.

$$0.8p - (0.2)(.8p) = 0.64p$$

4. **(A)** The survey size of 80 is sizable enough to draw conclusions about the class in general. Since 15.6% of the surveyed students felt that additional crosswalks were necessary, that fraction of the entire class will probably feel the same way.

Find 15.6% of 320.

$$320(0.156) = 49.92$$

Round the answer to 50 students who can be expected to favor additional crosswalks.

5. **(C)** Find the percent increase of hits from 2021 to 2022.

$$\frac{\text{increase}}{\text{original}} = \frac{x}{100}$$

Find the increase from 2021 to 2022.

$$3,230 - 2,150 = 1,080$$

$$\frac{1,080}{2,150} = \frac{x}{100}$$

$$x = 50.2\%$$

Steve believes the 50.2% increase is triple the projected increase from 2022 to 2023. Thus, divide 50.2% by 3 to get the expected increase from those years.

$$50.2\% \div 3 = 16.7\%$$

Multiply the 2022 figure by 1.167 to find the expected hits on the website in 2023.

$$3,230 \times 1.167 = 3,769$$

Choice (C), 3,770, is closest to this answer.

6. **(B)** Find the number of servings of roasted asparagus that provide the same nutritional value of vitamin C as one serving of a red bell pepper.

$$123\% \div 12\% = 10.25$$

It takes 10.25 servings of roasted asparagus to equal the amount of vitamin C in one serving of red bell pepper. Multiply the grams in one serving of roasted asparagus, 134, by 10.25.

$$134 \times 10.25 = 1,373.5$$

Choice (B), 1,400, is closest to this value.

7. **(A)** Satisfying the minimum daily requirement means 100% of that nutrient has been consumed in a day. Since one serving of roasted asparagus provides 16% of the daily iron requirement, divide 100% by 16% to determine the number of servings needed each day.

$$100 \div 16 = 6.25$$

A person needs 6.25 servings of roasted asparagus to satisfy the minimum daily requirement of iron. Multiply 6.25 by 134 to find the number of grams of roasted asparagus needed to reach this goal.

$$134 \times 6.25 = 837.5$$

Of the selections provided, choice (A), 800, is closest to this value.

8. **66** Although the suggested consumption of potassium for men is 4,700 milligrams, Jake's daily running indicates he needs an additional 10% above the norm. Multiply 4,700 by 1.1 to find the potassium requirement for Jake because of his running.

$$4,700 \times 1.1 = 5,170$$

Jake currently consumes 3,200 milligrams, so he needs an additional 1,970 milligrams. By adding an extra potato and a cup of lima beans, he will add  $1,081 + 955 = 2,036$  milligrams of potassium. Since he needs 1,970 milligrams, the addition of these new foods will provide  $2,036 - 1,970 = 66$  milligrams more than is necessary.

9. **50** The supermarket chain manager wants to earn an 80% customer service rating from its first 20 customer service surveys. Multiply 20 by 80 to get 1,600, the total number of additional percentage points to be amassed. We found that the first 10 reviews yielded an average score of 65% or  $10 \times 65 = 650$  percentage points. By subtracting 650 from 1,600 we get 950, the total number of percentage points to be amassed to earn the overall 80% rating. The question asks what could be the lowest the 17th survey could earn and still achieve the goal of 80% customer satisfaction from the first 20 surveys. Suppose all of the surveys from 11 through 16 and 18 through 20 gave the chain a score of 100%. That would mean those 9 customers added 900 points to the total. Given that we need 950 points from the final 10 customers, the 17th survey could score the chain as low as 50% and the chain would still receive an 80% average from the first 20 surveys.
10. **(D)** If  $n$  is the largest of the three numbers, then  $780 - n$  is the sum of the other two. Since  $n$  is 50% greater than the sum of the smaller numbers, we arrive at the following.

$$n = 1.5(780 - n)$$

$$n = 1,170 - 1.5n$$

$$2.5n = 1,170$$

$$n = 468$$

$$(0.6)(780) = 468$$

## Surveys Drill

For multiple-choice questions, solve the problem and pick the correct answer from the provided choices. Each multiple-choice question has only one correct answer. For student-produced response questions, solve each problem and write down your answers separately.

1. Sammie has done a survey of one hundred 16-year-old Florida students and found that 84% favored including an internship in their schools' curricula. She finds, however, that the margin of error in her survey is 6.9%. What can she do to reduce the margin of error?
  - (A) Check similar surveys in other states
  - (B) Reduce the survey size
  - (C) Increase the survey size
  - (D) Survey other age groups as well
2. Pham is interested in starting a 4-H Club at her high school. She randomly sampled a group of 200 students and found that 31.5% of the students in her sample thought that starting a club was a good idea. If her high school has 1,145 students enrolled, approximately how many will consider that starting the 4-H Club was a good idea?
  - (A) 360
  - (B) 340
  - (C) 280

(D) 225

3. Shameka took a survey of 90 randomly chosen tenth graders to see if they would prefer a year-round school year. The survey results showed that approximately 41.1% of the tenth graders preferred the year-round schedule. If there are 245 tenth graders at the school, how many can be expected to prefer the year-round schedule? (Round your answer to the nearest student.)

(A) 101

(B) 100

(C) 96

(D) 86

4. A survey of 150 randomly chosen adults in Amarillo, Texas, found that 13.2% prefer electric heat over gas heat. If there are 111,115 adults in Amarillo, how many adults will not prefer electric heat?

(A) 14,667

(B) 36,561

(C) 96,448

(D) 111,115

5. In a survey of 180 randomly chosen dog owners, 54 preferred Daisy's Dog Treats over another brand. If city A, where the survey was taken, has 4,840 dog owners, approximately how many would likely prefer Daisy's Dog Treats?

(A) 3,388

(B) 2,447

(C) 1,467

(D) 1,452

6. At a busy park, 124 people were surveyed to discern if additional city funds should be provided for additional parks. Of the 124 surveys, 11 chose not to respond. Which factor indicates the survey results will not be reliable?

(A) The weather on the day the survey was given

- (B) The percentage of surveys that showed a response
  - (C) The number of surveys that were returned unanswered
  - (D) Where the survey was given
7. Maria conducted a survey of people employed in New York's fashion industry. She was disappointed that the margin of error in the results was over 7%. What can Maria do in the next survey to reduce the margin of error by 3%?
- (A) Change 40% of the survey questions
  - (B) Conduct the survey in another city
  - (C) Increase the number of survey participants
  - (D) Decrease the number of survey participants
8. Samuel conducted a survey of local Boys Club members regarding the need for a new pool. He was delighted to find that the survey clearly predicted the actual results: 76 of the 171 members did not want a new pool. If Samuel interviewed 50 members, which of the following is closest to the number of respondents who did not prefer a new pool?
- (A) 21
  - (B) 22
  - (C) 29
  - (D) 39
9. A manufacturer of hearing aids conducted a study of 200 randomly selected customers. The first group of 100 customers, section A, felt that a new type of hearing aid improved the wearer's hearing by 70%. The second group of 100 customers, section B, used an older style of hearing aid and saw little improvement in their hearing. Which of the following can be concluded about the hearing aid study?
- (A) Hearing aids used by section B might not work.
  - (B) The users in section A and section B will notice similar improvement in their hearing.
  - (C) Those who use the new style of hearing aid from section A will hear better than those in section B.
  - (D) No useful conclusions can be drawn from the study.

10. Pollsters predict that Candidate A will be victorious in an upcoming election by garnering 54% of the vote. The margin of error in the survey is  $\pm 2.5\%$ . What estimate  $n$  will the pollsters expect for Candidate A if 150,000 votes are cast?
- (A)  $79,000 < n < 81,000$
  - (B)  $80,100 < n < 81,750$
  - (C)  $118,725 < n < 122,450$
  - (D)  $78,975 < n < 83,025$

### Answer Explanations

1. **(C)** If Sammie interviewed every 16-year-old in Florida, she would have no margin of error: she would have all of the data! Thus, the larger the sample size, the smaller the margin of error. Sammie ought to interview additional students to reduce the margin of error in the sample.
2. **(A)** Because Pham surveyed a random sample of the students in her school, her sample was representative of the entire high school. Thus, the percentage of students in the entire high school expected to prefer starting a 4-H Club is appropriately estimated by the percentage of students who preferred it in the sample, 31.5%. Thus, of the 1,145 students in the high school, approximately  $1,145 \times 0.315 = 360.7$  students would be expected to prefer starting the 4-H Club. Of the choices given, this is closest to 360.
3. **(A)** The survey respondents were randomly chosen, so the results among the tenth graders were unbiased. Therefore, multiply the total class size, 245, by 41.1%.

$$(245)(.411) = 101$$

4. **(C)** If 13.2% of the adults in Amarillo prefer electric heat, then 86.8% do not prefer electric heat.

$$100\% - 13.2\% = 86.8\%$$

Multiply 111,115, the number of adults living in Amarillo, by 86.8%.

$$(111,115)(0.868) \approx 96,448$$

Answer choice (A) refers to the Amarillo adults who prefer electric heat over gas heat.

5. **(D)** The survey respondents were randomly chosen, so the results were unbiased. Multiply 4,840 by  $\frac{54}{180}$ .

$$\left(\frac{54}{180}\right)(4,840) \approx 1,452$$

6. **(D)** People who use parks have a bias; they enjoy parks and would likely appreciate additional funds being appropriated for more parks.
7. **(C)** If Maria increased the size of the survey respondents, the margin of error would decrease. Theoretically, if Maria surveyed everyone in the fashion industry in New York, the margin of error would be 0%: she'd have all of the data!
8. **(B)** Construct a proportion to find the number of respondents who did not want a new pool. Use this model.

$$\frac{\text{those who don't want a pool}}{\text{total}} = \frac{\text{those who don't want a pool}}{\text{total}}$$
$$\frac{x}{50} = \frac{76}{171}$$

Cross-multiply and solve for  $x$ .

$$(171)(x) = (50)(76)$$

$$171x = 3,800$$

$$x \approx 22.222\dots$$

Choice (B), 22, is closest to this result.

9. **(C)** Given that users of the new hearing aid heard significantly better, we can conclude that the new hearing aid improves hearing. Although

we can also conclude that the older style of hearing aid is less effective than the new style, this fact was not included in the answer choices.

10. **(D)** Find the predicted number of votes earned by Candidate A.

$$(150,000)(0.54) = 81,000$$

The margin of error is  $\pm 2.5\%$ , so add and subtract 2.5% of 81,000 to and from 81,000.

$$81,000 - (0.025)(81,000) = 78,975$$

$$81,000 + (0.025)(81,000) = 83,025$$

The range of votes collected by Candidate A should fall in the following range.

$$78,975 < n < 83,025$$

## Graphs and Data Interpretation

### Drill 1

For multiple-choice questions, solve the problem and pick the correct answer from the provided choices. Each multiple-choice question has only one correct answer. For student-produced response questions, solve each problem and write down your answers separately.

### Table of the Seven Lightest Chemical Elements (by atomic mass)

Atomic Mass	Name of Chemical Element	Symbol	Atomic Number
1.0079	Hydrogen	H	1
4.0026	Helium	He	2
6.941	Lithium	Li	3
9.0122	Beryllium	Be	4
10.811	Boron	B	5
12.0107	Carbon	C	6
14.0067	Nitrogen	N	7

1. Based on the table above, the atomic mass of which element is approximately equal to the sum of the atomic masses of helium and lithium?
- (A) Nitrogen  
 (B) Boron  
 (C) Beryllium  
 (D) Lithium

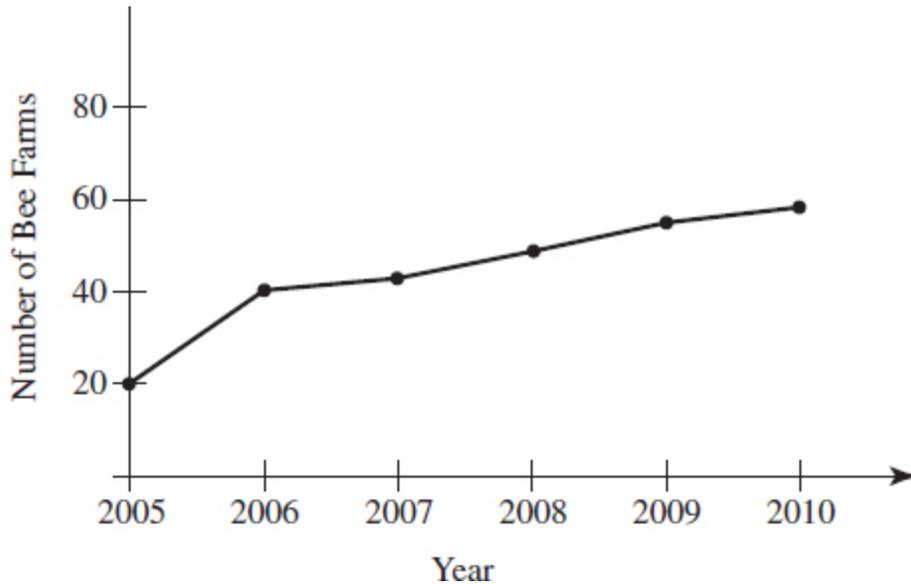
**Table of the Seven Lightest Chemical Elements (by atomic mass)**

Atomic Mass	Name of Chemical Element	Symbol	Atomic Number
1.0079	Hydrogen	H	1
4.0026	Helium	He	2
6.941	Lithium	Li	3
9.0122	Beryllium	Be	4
10.811	Boron	B	5
12.0107	Carbon	C	6
14.0067	Nitrogen	N	7

2. Based on the table above, which element is about 30% more massive than lithium?
- (A) Nitrogen  
 (B) Boron

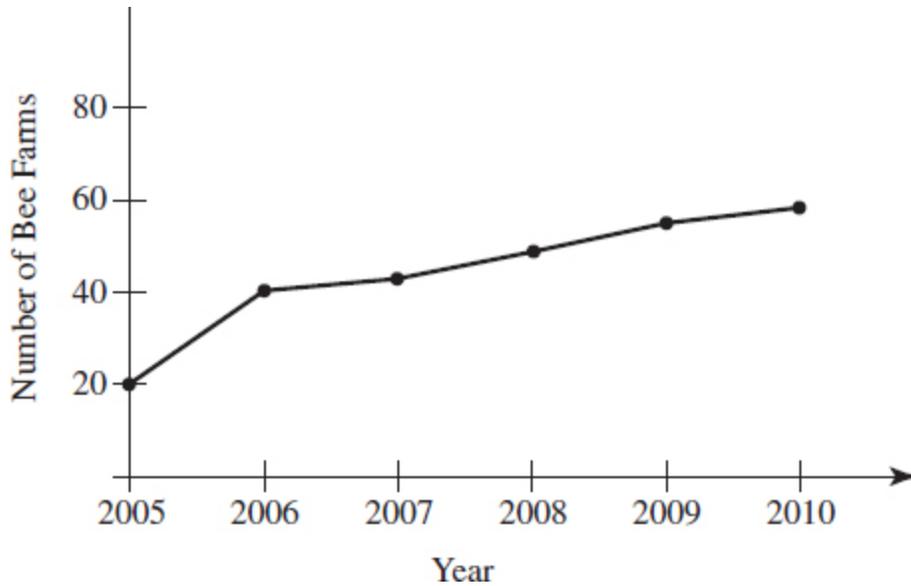
- (C) Beryllium
- (D) Hydrogen

3. The graph below shows the number of domestic bee farms in the metropolitan Atlanta area from 2005 to 2010.



Based on the graph, which of the following describes the trend in the number of domestic bee farms in the Atlanta area?

- (A) Exponential growth between 2005 and 2010
  - (B) Continued but modest growth after 2006
  - (C) An increase until 2007 followed by a gradual decline after that year
  - (D) A dramatic decrease after 2008
4. The graph below shows the number of domestic bee farms in the metropolitan Atlanta area from 2005 to 2010.



If each bee farm harvests 11,000 gallons of honey annually, which of the following is closest to the number of gallons collected in 2006?

- (A) 110,000
- (B) 220,000
- (C) 440,000
- (D) 660,000

5. The following table represents federal spending for NASA for the years 1958 through 1963.

**NASA Spending Since 1958**

Year	NASA Fed Outlay, \$ in Millions	Total U.S. Fed Spending, \$ in Millions	NASA as % of U.S. Spending	President	Party
1958	89	71,936	0.1	Dwight D. Eisenhower	Rep
1959	145	80,697	0.2	Dwight D. Eisenhower	Rep
1960	401	76,539	0.5	Dwight D. Eisenhower	Rep
1961	744	81,515	0.9	John F. Kennedy	Dem
1962	1,257	106,821	1.18	John F. Kennedy	Dem
1963	2,552	111,316	2.28	Lyndon B. Johnson	Dem

Which of the following can be concluded about the information portrayed in the table?

- (A) From 1958 until 1963, spending on NASA as a percentage of the U.S. budget increased nearly 2,200%.
- (B) Between 1962 and 1963, spending on NASA virtually tripled.
- (C) In 1963, real spending on NASA exceeded the \$1 billion mark for the first time.
- (D) The total dollars spent each year from 1958 to 1963 decreased.

6. The following table represents federal spending for NASA for the years 1958 through 1963.

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1962	1,257	106,821	1.18	John F. Kennedy	Dem
1963	2,552	111,316	2.28	Lyndon B. Johnson	Dem

Total U.S. spending in 1962 was about how many times larger than the NASA budget in 1958?

- (A) 77 times
- (B) 88 times
- (C) 650 times
- (D) 1,200 times

7. The table below is a statistics page for an educational website. The visits, files, and other data are shown for the first week of June 2015. The owner of the educational site is wondering how to effectively reach more viewers.

### Daily Statistics for June 2015

Day	Hits		Files		Pages		Visits		Sites		KBytes	
1	327	6.35%	274	6.62%	250	7.29%	85	7.58%	60	8.97%	2203	4.87%
2	432	8.39%	383	9.25%	349	10.17%	64	5.71%	57	8.52%	2506	5.54%
3	283	5.49%	235	5.68%	181	5.28%	56	5.00%	50	7.47%	2332	5.15%
4	249	4.83%	208	5.03%	143	4.17%	47	4.19%	56	8.37%	2816	6.22%
5	265	5.14%	164	3.96%	134	3.91%	39	3.48%	44	6.58%	1644	3.63%
6	165	3.20%	142	3.43%	114	3.32%	45	4.01%	42	6.28%	1425	3.15%
7	250	4.85%	155	3.74%	134	3.91%	50	4.46%	51	7.62%	1621	3.58%

Which of the following can be concluded from the table?

- (A) The number of visits exceeds the number of sites each day.
  - (B) On day 7, the ratio of hits to files is greater than the ratio of files to pages.
  - (C) The ratio of KBytes to sites is greater on day 6 than on day 1.
  - (D) The ratio of pages to visits is never less than 3:1.
8. The table below is a statistics page for an educational website. The visits, files, and other data are shown for the first week of June 2015. The owner of the educational site is wondering how to effectively reach more viewers.

### Daily Statistics for June 2015

Day	Hits		Files		Pages		Visits		Sites		KBytes	
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7	250	4.85%	155	3.74%	134	3.91%	50	4.46%	51	7.62%	1621	3.58%

What is the median number of visits in the first seven days of June 2015?

- (A) 85
- (B) 47
- (C) 56
- (D) 50

9. The table below is a statistics page for an educational website. The visits, files, and other data are shown for the first week of June 2015. The owner of the educational site is wondering how to effectively reach more viewers.

**Daily Statistics for June 2015**

Day	Hits		Files		Pages		Visits		Sites		KBytes	
1	327	6.35%	274	6.62%	250	7.29%	85	7.58%	60	8.97%	2203	4.87%
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6	165	3.20%	142	3.43%	114	3.32%	45	4.01%	42	6.28%	1425	3.15%
7	250	4.85%	155	3.74%	134	3.91%	50	4.46%	51	7.62%	1621	3.58%

If  $R$  is the range of the Files category and  $r$  is the range of the KBytes category, what is the value of  $2r - R$ ?

- (A) 3,126
- (B) 2,541
- (C) 254
- (D) -249

Age	Target Heart Rate Zone, 50–85%	Average Maximum Heart Rate, 100%
-----	--------------------------------	----------------------------------

Age	Target Heart Rate Zone, 50–85%	Average Maximum Heart Rate, 100%
20 years	100–170 beats per minute	200 beats per minute
30 years	95–162 beats per minute	190 beats per minute
35 years	93–157 beats per minute	185 beats per minute
40 years	90–153 beats per minute	180 beats per minute

10. The table above indicates data relating to heartbeat rates for adults between the ages of 20 and 40 as put forth by the American Medical Association. If a woman age 35 has a heart rate that is the mean of the extremes of the target heart rate zone for her age, what percent of the maximum heart rate is that measure? (Round to the nearest tenth of a percent.)

### Answer Explanations

- (B)** The combined atomic masses of helium and lithium are  $4.0026 + 6.941 = 10.9436$ . The atomic mass of boron is 10.811. That value is the closest to the combined atomic masses of helium and lithium among the answer choices.
- (C)** The atomic mass of lithium is 6.941. Multiply that number by 0.30 and add it to the atomic mass of lithium.

$$6.941 + (0.30)(6.941) = 9.023$$

Of the elements listed in the table, beryllium, with an atomic mass of 9.0122, is approximately 30% more massive than lithium.

- (B)** From 2005 to 2006, there was a 100% growth (from 20 to 40). After 2006, however, the graph shows about a 5% increase per year.

4. **(C)** The graph indicates that approximately 40 bee farms were operating in 2006. Multiply 40 by 11,000 to find the 2006 yield of honey.

$$40 \times 11,000 = 440,000$$

5. **(A)** The percentage of the U.S. budget spent on NASA was 0.1% in 1958. By 1963, the percentage of the U.S. budget spent on NASA was 2.28%. Use the formula  $\frac{\text{increase}}{\text{original}} = \frac{n}{100}$  to find the percent increase.

$$\text{Increase} = 2.28 - 0.1 = 2.18$$

$$\frac{2.18}{1} = \frac{n}{100}$$

$$218 = 0.1n$$

$$n = 2,180\%$$

Choice (A), 2,200%, is the closest to this figure.

6. **(D)** Total U.S. spending in 1962 was \$106,821,000,000. The NASA budget in 1958 was \$89,000,000. Divide 106,821,000 by 89,000,000 to calculate how many times larger the 1962 U.S. spending was than the 1958 NASA budget.

$$106,821,000,000 \div 89,000,000 = 1,200.23$$

Choice (D), 1,200 times larger, is the closest.

7. **(B)** On day 7, the ratio of hits to files is greater than the ratio of files to pages. Note the ratio of hits to files.

$$\frac{250}{155} \approx 1.61$$

The ratio of files to pages is  $\frac{155}{134} \approx 1.16 \approx 1.16$ .

$$1.61 > 1.16$$

Choices (A), (C), and (D) cannot be supported by the data in the table.

8. **(D)** The median is the middle value when the numbers are arranged in order. The numbers are arranged in order below.

39 45 47 50 56 64 85

The value in the middle, 50, is the median.

9. **(B)** The range of a group of numbers is found by finding the difference between the greatest and least values.

The range of the Files category is  $383 - 142 = 241$ .

$$R = 241$$

The range of the KBytes category is  $2,816 - 1,425 = 1,391$ .

$$r = 1,391$$

Find  $2r - R$  by substituting 241 and 1,381 for  $R$  and  $r$ , respectively.

$$(2)(1,391) - 241 = 2,541$$

10. **67.6** A woman age 35 has a target heart rate zone of 93 to 157 beats per minute. Find the mean of that range by adding  $93 + 157$  and dividing by 2.

$$93 + 157 = 250$$

$$250 \div 2 = 125$$

Find the percent that 125 is of 185 by dividing 125 by 185.

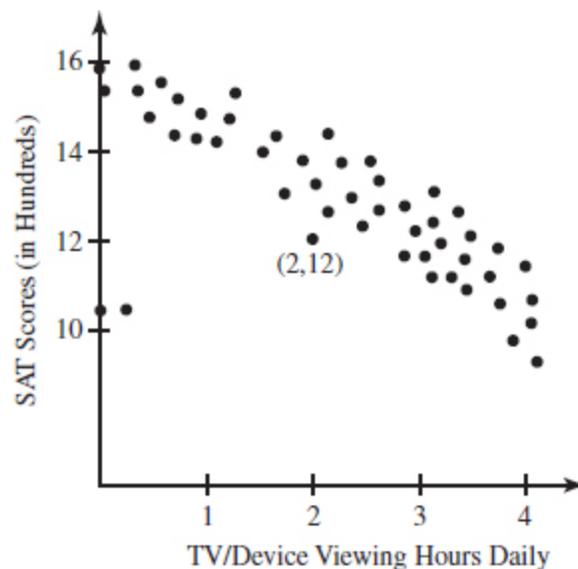
$$125 \div 185 = 0.6757$$

Rounded to the nearest tenth of a percent, 0.6757 is 67.6%. A heart rate of 125 is 67.6% of the maximum heart rate.

## Drill 2

For multiple-choice questions, solve the problem and pick the correct answer from the provided choices. Each multiple-choice question has only one correct answer. For student-produced response questions, solve each problem and write down your answers separately.

1. The advent of television, iPhones, and tablets is vying for students' study time. Experts are concerned that the prolonged use of electronic devices adversely affects SAT scores. The graph below demonstrates the correlation between hours spent viewing daily and their effect on SAT scores.

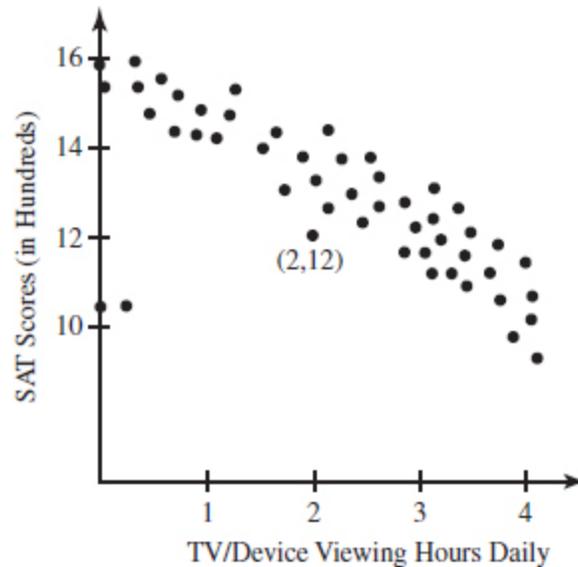


Which of the following can be inferred from the graph?

- (A) The impact of viewing 3 versus 4 hours resulted in the same SAT score.
- (B) All students who watched 1 hour or less of television/devices daily scored 1200 or higher on the SAT.
- (C) SAT scores varied inversely with daily television/device viewing.

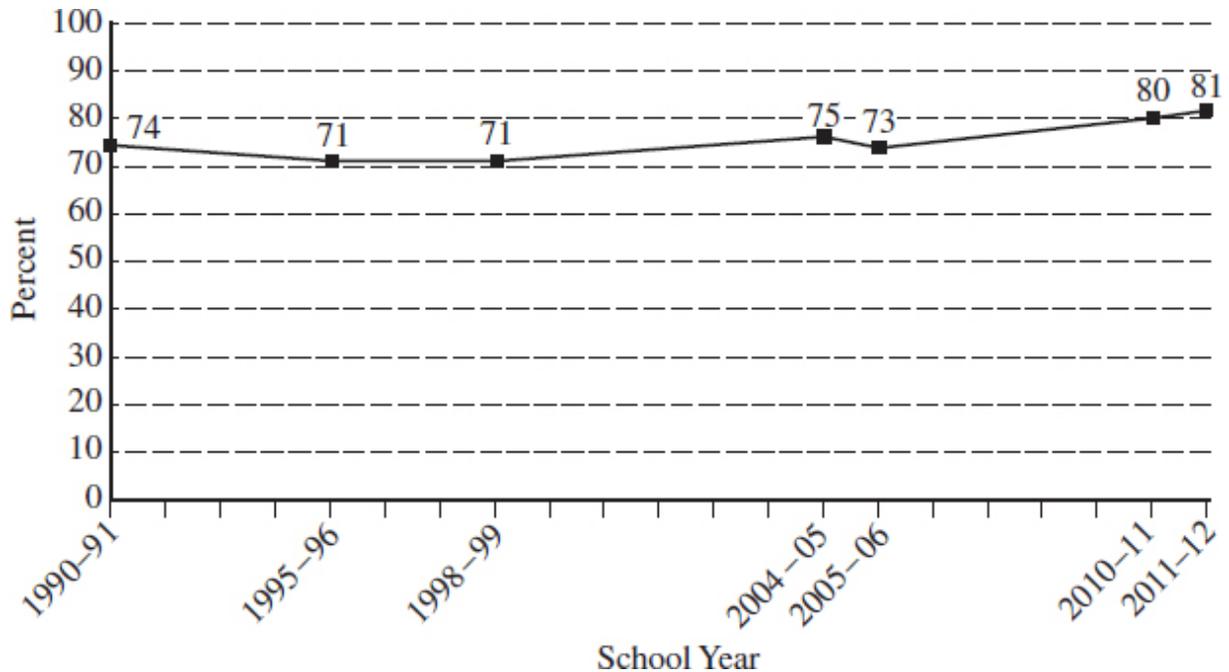
(D) SAT scores varied directly with daily television/device viewing.

2. The advent of television, iPhones, and tablets is vying for students' study time. Experts are concerned that the prolonged use of electronic devices adversely affects SAT scores. The graph below demonstrates the correlation between hours spent viewing daily and their effect on SAT scores.



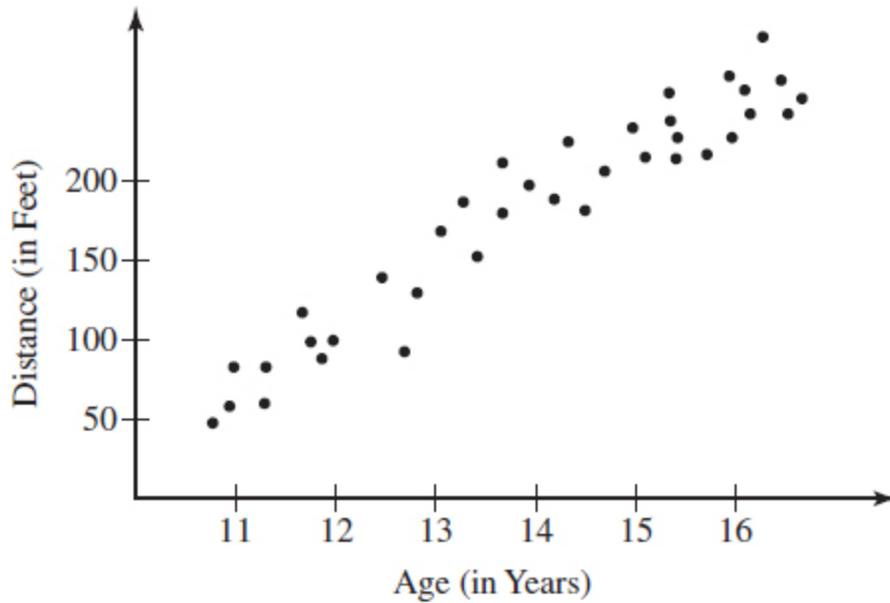
Given the data in the graph, which equation could be the line of best fit?

- (A)  $y = 2.3x - 4.8$
  - (B)  $y = 0.86x + 0.23$
  - (C)  $y = -6.2x - 5.9$
  - (D)  $y = -2.04x + 15.8$
3. The graph below reflects the average percentage of freshman high school students in America who graduate from high school within four years.



Which of the following can be inferred from the graph above?

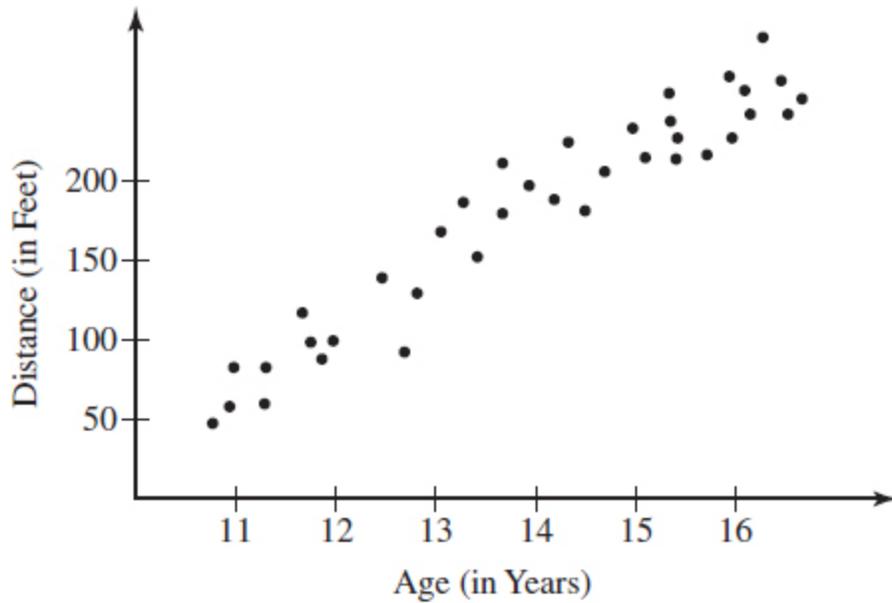
- (A) From 2006 to 2012 there was approximately an 11% increase in the percentage of high school graduates.
  - (B) There was a steady increase in the number of high school graduates between 1991 and 2012.
  - (C) The number of high school graduates decreased from 2006 to 2012.
  - (D) The state of the national economy had a marked increase on the percentage of high school graduates.
4. A girls' softball coach was recruiting for Behrens High School's spring softball team. The coach wanted to see if age was a determining factor in gauging the distance each girl could throw a softball.



Softball Throws by Age and Distance

Which of the following can be concluded from the graph?

- (A) There is a weak negative correlation between age and distance thrown.
  - (B) There is a strong positive correlation between age and distance thrown.
  - (C) There is no correlation between age and distance thrown.
  - (D) There is an inverse relationship between age and distance thrown.
5. A girls' softball coach was recruiting for Behrens High School's spring softball team. The coach wanted to see if age was a determining factor in gauging the distance each girl could throw a softball.

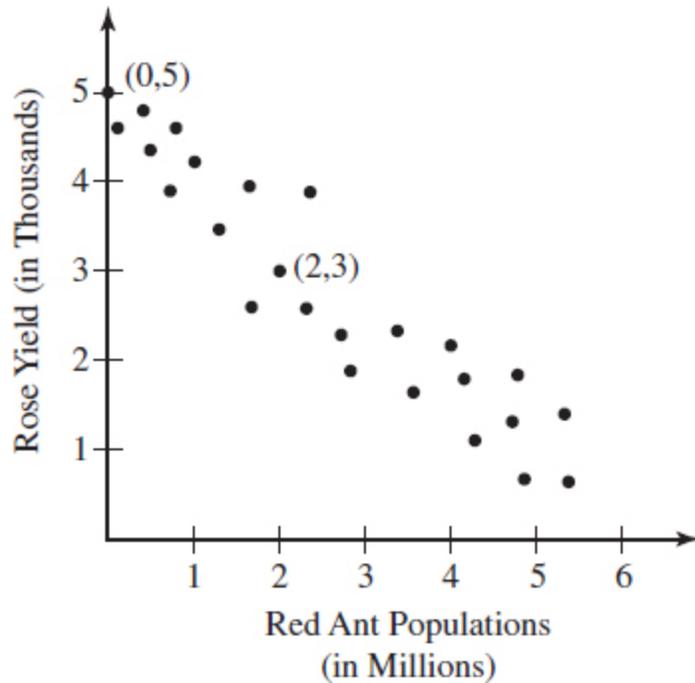


Softball Throws by Age and Distance

Which of the following functions is a line of best fit for the graph?

- (A)  $y = 5.14x - 123.5$
- (B)  $y = -43.27x - 231.3$
- (C)  $y = 33.33x - 299.6$
- (D)  $y = 14.2 - 2.01x$

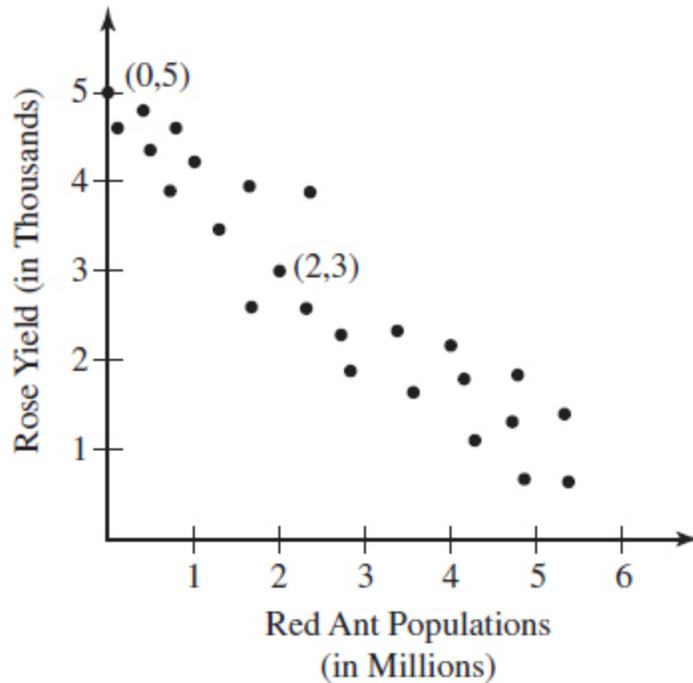
6. A botanist was exploring the impact a local red ant population had on local nurseries. An experiment was devised to investigate any correlation between red ant populations and the annual yield of white roses. A scatter plot of the data is shown.



A line of best fit is constructed within the data. Which of the following functions best models the data?

- (A)  $f(x) = -4.88x + 4.66$
- (B)  $f(x) = 1.15x + 7.88$
- (C)  $f(x) = 1.27x + 4.88$
- (D)  $f(x) = -1.14x + 4.88$

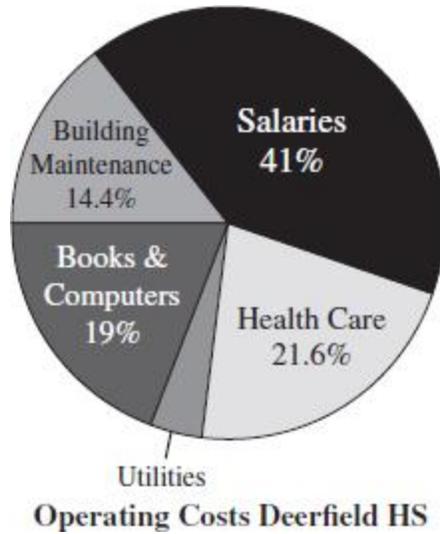
7. A botanist was exploring the impact a local red ant population had on local nurseries. An experiment was devised to investigate any correlation between red ant populations and the annual yield of white roses. A scatter plot of the data is shown.



A grower uses a pesticide that will reduce the ant population by 60%. If the grower currently has five million ants, which of the following is the closest to his rose yield after using the pesticide?

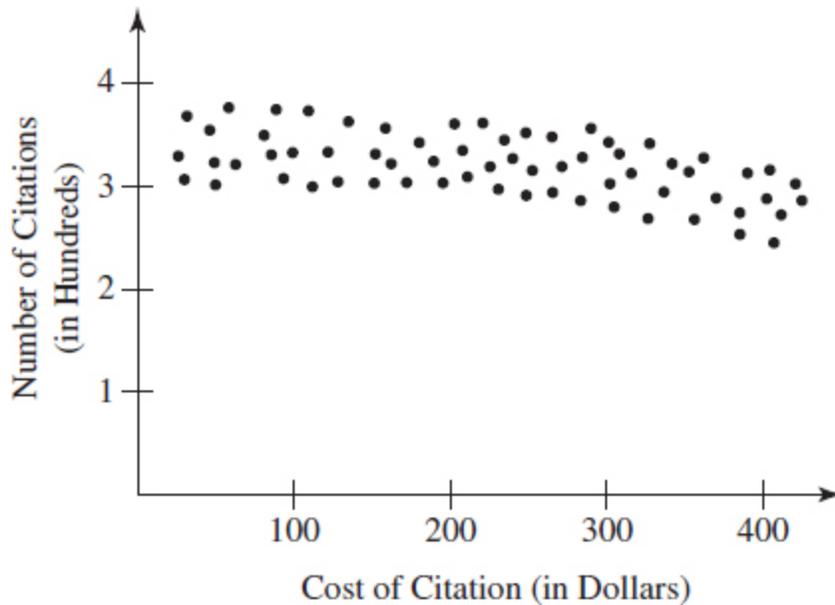
- (A) 1,000
- (B) 1,800
- (C) 3,000
- (D) 4,200

8. The pie chart below represents the operating costs for Deerfield High School in 2014.



The cost for health care benefits in 2014 is \$4,100,000. What is the cost for utilities?

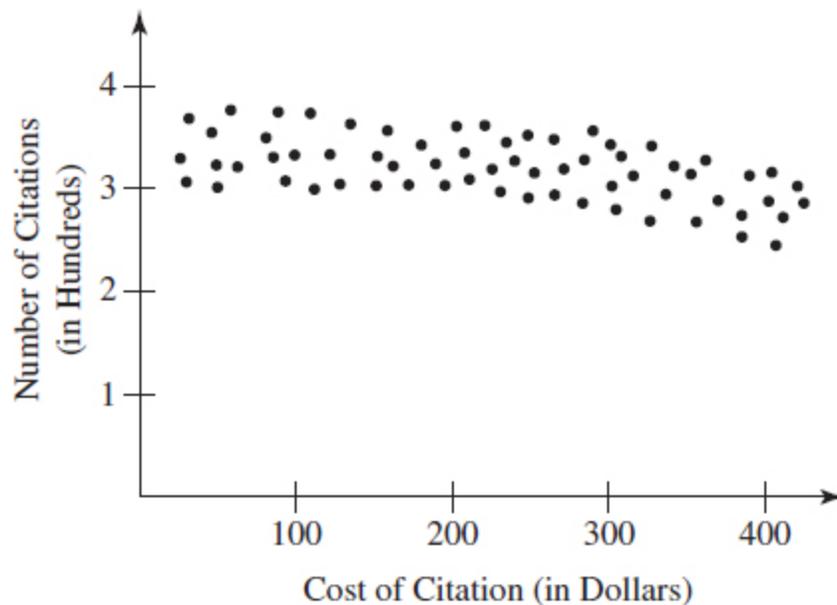
- (A) \$586,113
- (B) \$637,431
- (C) \$759,259
- (D) \$817,437



9. The graph shows the number of drivers who have been caught texting while driving and the fines that were levied against them. Which of

the following can be concluded from the graph?

- (A) There is a strong negative correlation between higher fines and the number of drivers cited for texting while driving.
- (B) There is a weak negative correlation between higher fines and the number of drivers cited for texting while driving.
- (C) There is a strong positive correlation between higher fines and the number of drivers cited for texting while driving.
- (D) There is no correlation between higher fines and the number of drivers cited for texting while driving.



10. If the fine for texting while driving was increased to \$800, approximately how many citations could be expected to be issued?
- (A) 70
  - (B) 230
  - (C) 460
  - (D) 520

### Answer Explanations

1. **(C)** Inverse variation means one variable increases proportionally to the decrease in the other variable. SAT scores generally dropped as

daily television/device watching increased.

2. **(D)** The data appear to descend to the right, indicating a negative slope. Therefore, choices (A) and (B) can be eliminated. Choose two points in the center of the data, and derive the equation of the line that connects them.

$$\begin{aligned} & (1, 14) \quad (2, 12) \\ m &= \frac{y_2 - y_1}{x_2 - x_1} \\ m &= \frac{12 - 14}{2 - 1} \\ m &= -2 \\ y &= -2x + b \end{aligned}$$

Substitute either point for  $x$  and  $y$ .

$$\begin{aligned} 14 &= -2(1) + b \\ 16 &= b \end{aligned}$$

The equation of a line of best fit could be  $y = -2x + 16$ . Choice (D) is the closest to this equation.

3. **(A)** The percentage of high school graduates in 2006 was 73%, and in 2012 the percentage had increased to 81%. Divide 81 by 73 to find the percent increase.

$$81 \div 73 = 1.109$$

The increase was 10.9%; the closest answer to this figure is 11%.

4. **(B)** The graph shows an unmistakable upward rise to the right; as the age increases, so does the distance thrown. Choice (A), which indicates a weak negative correlation in the graph, can be discarded; the graph clearly moves vertically as age increases.
5. **(C)** Find the line of best fit by selecting two points that lie within the data.

$$(12, 100) (15, 200)$$

Find the slope of a line that connects the two points.

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{200 - 100}{15 - 12} \approx 33.33$$
$$y = 33.33x + b$$

Use either point to find  $b$ .

$$100 = 33.33(12) + b$$
$$-299.6 = b$$
$$y = 33.33x - 299.6$$

6. **(D)** The data roughly correspond to a line with a negative slope, so only choices (A) and (D) should be considered. By using the points (0, 5) and (2, 3), we can get a sense of the slope:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 3}{0 - 2} = -1$$

Choice (D) is more closely aligned with a slope of  $-1$ . Choices (B) and (C) can be eliminated at once. The data appear to descend to the right as the number of ants increases, thus indicating a negative slope. Choices (B) and (C) incorrectly show positive slopes.

7. **(C)** After the grower uses the pesticide, his ant population will be reduced by 60%. Find 60% of 5,000,000 and subtract that amount from 5,000,000.

$$5,000,000 - (0.60)(5,000,000) = 2,000,000$$

The graph indicates the point (2, 3) is on the line of best fit. Thus, when the grower has 2,000,000 ants, he can expect a yield of 3,000 roses.

8. **(C)** Find the percentage of operating costs for utilities in 2015.

$$100\% - (14.4 + 41 + 21.6 + 19) = 4\%$$

Set up a proportion comparing an item's percentage of the budget to its dollar expenditure.

$$\frac{21.6}{4,100,000} = \frac{4}{x}$$

Cross-multiply and solve for  $x$ .

$$(21.6)(x) = (4)(4,100,000)$$

$$21.6x = 16,400,000$$

$$x = \$759,259$$

9. **(B)** Although the number of drivers cited for texting while driving is reduced slightly as higher fees are levied, a line of best fit would show only a mild negative slope. Thus, there is a weak negative correlation between higher fines and the number of drivers cited for texting while driving.
10. **(B)** The data points in the graph indicate a modest decrease in the number of citations issued as the cost of fines increases. Thus, we expect the number of citations to be fewer than 300. We can eliminate answer choices (C) and (D).

Pick two data points to find a line of best fit, such as (100, 350) and (400, 300).

Start to create an equation for the line of best fit by first finding the slope.

$$m = \frac{300 - 350}{400 - 100} = -\frac{1}{6}$$

$$y = -\frac{1}{6}x + b$$

Select one of the points to calculate  $b$ , the  $y$ -intercept.

$$300 = -\frac{1}{6}(400) + b$$

$$367 \approx b$$

$$y = -\frac{1}{6}x + 367$$

Replace  $x$  with 800 to estimate the number of citations issued when the fine is \$800.

$$y = -\frac{1}{6}(800) + 367$$

$$y = 233$$

Choice (B), 230, is closest to this estimate.

## Probability Drill

For multiple-choice questions, solve the problem and pick the correct answer from the provided choices. Each multiple-choice question has only one correct answer. For student-produced response questions, solve each problem and write down your answers separately.

Fish	6–12 inches	13–24 inches	Total
Bullhead	26	18	44
Trout	31	47	78
Mullet	53	44	97
Total	110	109	218

1. A popular fishing site recorded the number of fish caught on a particular day as shown in the table above. If a certain fish caught that day was 11 inches long, what is the probability that the fish was a mullet?

- (A) 0.242
- (B) 0.316
- (C) 0.482
- (D) 0.546

2. A game spinner has four colors equally spaced: red, green, blue, and yellow. What is the probability that a single, six-sided die will land on a prime number followed by a spin of yellow or blue?

- (A)  $\frac{1}{4}$
- (B)  $\frac{1}{9}$
- (C)  $\frac{1}{12}$
- (D)  $\frac{1}{36}$

	10th	11th
Female	17	18
Male	18	20

3. One lucky student at Bristol High School will be selected to attend a College for Kids camp at Johns Hopkins University. The table above represents the number of students in the 10th and 11th grades at the school. If a student is selected at random, what is the probability that the student will be in 10th grade or a male?

- (A) 0.753
- (B) 0.521
- (C) 0.479
- (D) 0.472

4. In a 10-question true/false test, what is the probability of guessing correctly on exactly two of the questions from 6 through 8?

- (A)  $\frac{1}{27}$
- (B)  $\frac{1}{3}$
- (C)  $\frac{1}{3}$

(D)  $\frac{7}{8}$

	7th Grade	8th Grade	9th Grade
Girls	191	215	174
Boys	162	219	181

5. The data above represent students categorized by gender and grade at a local middle school. If a student is selected at random, what is the probability the student is not an eighth-grade girl? (Round your answer to the nearest hundredth.)

	Boy	Girl	Total
Swim	12	14	26
Water Polo	16	15	31
Total	28	29	57

6. The above chart represents the students who are members of a high school's swim and water polo teams. The water polo and swim teams are having a Spring Banquet. One of the team members will be chosen as athlete of the year. What is the probability the person chosen will be a member of the water polo team or a boy?

- (A)  $\frac{31}{57}$   
(B)  $\frac{43}{57}$   
(C)  $\frac{52}{57}$   
(D)  $\frac{55}{57}$

7. Swimmers who participate in backstroke races depend on a series of banners strung overhead. The banners warn the swimmer that the wall is 5 yards away. A company sells these banners in long cords that repeat a sequence of colors in the following order:

Red Yellow Blue Green Orange Purple

If a high school bought 40 yards of a banner string that contained 75 banners, what was the color of the 59th banner?

- (A) Yellow
- (B) Green
- (C) Orange
- (D) Purple

8. In William's dresser are three pairs of white socks, five pairs of blue socks, and two pairs of brown socks. What is the probability of William selecting a brown pair, replacing it in the drawer, and then selecting a blue pair of socks?

- (A)  $\frac{1}{10}$
- (B)  $\frac{2}{19}$
- (C)  $\frac{3}{10}$
- (D)  $\frac{7}{10}$

9. Cindy has three pairs of white socks, five pairs of blue socks, and two pairs of brown socks in her dresser drawer. What is the probability of Cindy withdrawing a blue pair of socks and, without replacing it, drawing a pair of white socks? (Round your answer to the nearest thousandth.)

10. There is a 40% chance of rain on Thursday. If it does not rain, a high school baseball team has a 70% chance that it will play its scheduled game. What is the probability that there will not be any rain on Thursday but the game will not be played?

- (A) 14%
- (B) 18%
- (C) 28%
- (D) 42%

## Answer Explanations

1. **(C)** According to the table, 110 fish were caught that were 6–12 inches long. Of that amount, 53 were mullets. Divide 53 by 110 to find the

probability that a particular fish caught that measured 6–12 inches was a mullet.

$$\frac{53}{100} \approx 0.482$$

2. **(A)** Find the probability that a single, six-sided die will land on a prime number. Remember that 1 is not a prime number.

$$p(\text{prime number}) = \frac{2,3,5}{1,2,3,4,5,6} = \frac{1}{2}$$

Find the probability that the spinner will land on yellow or blue.

$$\frac{\text{yellow, blue}}{\text{red, green, blue, yellow}} = \frac{1}{2}$$

The two events, landing on a prime number and spinning yellow or blue, are independent. So multiply the probabilities.

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

3. **(A)** Use the probability formula.

$$P(x) = \frac{\text{desired outcomes}}{\text{all outcomes}}$$

There are 35 tenth graders and 38 boys. However, some of the tenth graders are boys and should not be counted twice. Thus:

$$P(\text{10th grader or boy}) = [(\text{number of 10th graders}) + (\text{number of boys}) - (\text{number of 10th-grade boys})] \div (\text{total number of students})$$

$$P(\text{10th grader or boy}) = (35 + 38 - 18) \div 73$$

$$P(\text{10th grader or boy}) = 55 \div 73 = 0.753$$

4. **(B)** A total of 8 outcomes are possible when guessing among 3 questions.

Let C = the answers guessed correctly

Let I = the answers guessed incorrectly

CCC, CCI, CIC, CII, ICC, ICI, IIC, III

Of the 8 possible outcomes, 3 contain exactly two correct guesses (and therefore one incorrect guess). Therefore, the probability of guessing correctly on exactly two of the three questions among questions 6 through 8 is  $\frac{3}{8}$ .

5. **.81** Find the probability that the student selected is an eighth-grade girl and subtract that value from 1.0.

$$\frac{\text{eighth-grade girl}}{\text{all students}} = \frac{245}{1,142} = 0.188$$

Subtract 0.188 from 1.0 to find the probability that the student selected is not an eighth-grade girl.

$$1.0 - 0.188 = 0.812$$

The answer, 0.812, rounds to 0.81.

6. **(B)** Find the probability that the athlete will be a boy or a water polo member. Avoid double counting by subtracting the boys who are on the water polo team.

$$\begin{aligned} \text{boys: } \frac{28}{57} \quad & \text{water polo members: } \frac{31}{57} \\ \text{boys on the water polo team: } & \frac{16}{57} \\ \frac{28}{57} + \frac{31}{57} - \frac{16}{57} &= \frac{43}{57} \end{aligned}$$

7. **(C)** The banners are sold in the same sequence of repeating colors:

Red Yellow Blue Green Orange Purple

There are 6 banners in each sequence, so divide 59 by 6 the same way you did in 4th grade.

$$59 \div 6 = 9 \text{ remainder } 5$$

The quotient is not important, but the remainder is. The quotient suggests that we have completed 9 full sequences and are now 5 deep into the 10th sequence of colors. Count 5 banners into the series to arrive at orange as the 59th banner.

8. **(A)** The probability of selecting a pair of brown socks is  $\frac{2}{10}$ . After replacing the pair, the probability of selecting a blue pair is  $\frac{5}{10}$ . Multiply the two fractions to find the probability of these two probabilities occurring sequentially.

$$\frac{2}{10} \times \frac{5}{10} = \frac{10}{100} = \frac{1}{10}$$

9. **.167 or .166** The probability of Cindy selecting a blue pair of socks is  $\frac{5}{10}$  or 0.5. The pair she selected is not replaced, so now there are nine pairs of socks in the drawer. The probability of Cindy selecting a white pair is  $\frac{3}{9}$  or 0.333. Multiply 0.5 by 0.333.

$$0.5 \times 0.333 \approx 0.167$$

10. **(B)** There is a 100% chance that rain will or will not occur on Thursday. Subtract 40% from 100% to find the probability of no rain.

$$100\% - 40\% = 60\%$$

There is a 100% chance that the game will or will not be played on Thursday if it does not rain. Subtract 70% from 100% to find the probability that the game will not be played, even if it does not rain.

$$100\% - 70\% = 30\%$$

Multiply the two percentages to find the probability of no rain on Thursday and the game not being played.

## Geometry and Trigonometry

The Geometry and Trigonometry portion of the digital SAT Math section will require the test taker to be proficient in the following topics:

### TIP

A majority of the geometry formulas will be provided in the reference section of each Math section. The savvy test taker should review these formulas before test day. However, the trigonometry formulas will not appear in this section, so we will show them here.

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- Area and volume
- Lines, angles, and triangles
- Right triangles and trigonometry
- Circles

The test taker needs to know the relationships of the sine, cosine, and tangent of a right triangle. The trigonometric ratios are as follows:

$$\text{sine} = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\text{cosine} = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

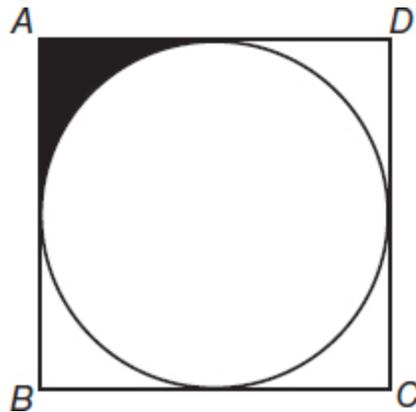
$$\text{tangent} = \frac{\text{opposite side}}{\text{adjacent side}}$$

The practice questions in this section will help you strengthen your skills in trigonometry.

### Area, Perimeter, and Volume Drill

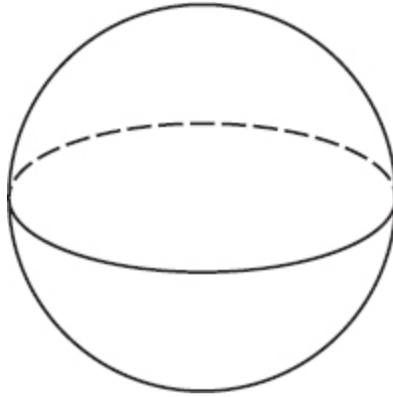
For multiple-choice questions, solve the problem and pick the correct answer from the provided choices. Each multiple-

choice question has only one correct answer. For student-produced response questions, solve each problem and write down your answers separately.



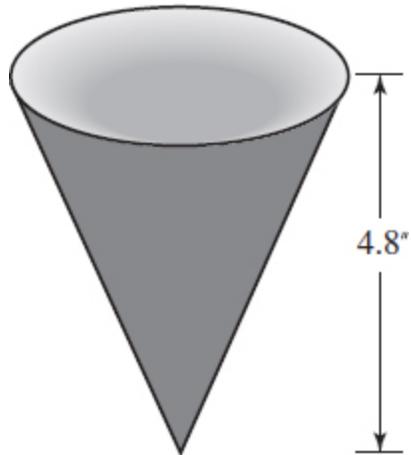
1. A coin toss board is shown above. If square  $ABCD$  has perimeter 64, what is the probability of a coin landing on the shaded portion of the board?  
  - (A) 0.363
  - (B) 0.182
  - (C) 0.098
  - (D) 0.053
2. An aquarium is in the shape of a right rectangular prism. The glass container has the following dimensions:  
length: 3 feet  
width: 2 feet  
  
The water in the tank rises to a height of 1 foot. The water is poured into another aquarium that has the following dimensions:  
length: 4 feet  
width: 1 foot  
  
If the water rises to a height of  $x$  feet in the second aquarium, what is the value of  $x$ ?

- (A) 1 foot
- (B) 1.5 feet
- (C) 2 feet
- (D) 2.5 feet

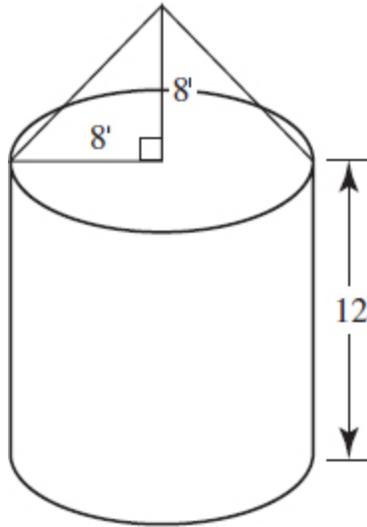


3. A great circle of a sphere contains the sphere's center. If the volume of a sphere is 374 cubic inches, what is the circumference of a great circle on that sphere? (Use  $\pi = 3.14$ )
- (A) 4.47
  - (B) 28.07
  - (C) 56.14
  - (D) 62.74
4. If the area of a rectangle is shown by the expression  $16x^2 - 9y^2$  and the width is  $4x - 3y$ , what is the perimeter of the rectangle?
- (A)  $4(x + y)$
  - (B)  $16x$
  - (C)  $64x^2 - 36y^2$
  - (D)  $2(x + y)$
5. A large snowball is in the shape of a sphere. At 10:00 A.M., the radius of the snowball is 2 feet. The radius grows 6 inches every 10 minutes it is rolled in the snow. If the snowball is continuously rolled, how much greater, in cubic feet, is the snowball at 11:30 A.M. than it is at 10:00 A.M.? (Round your answer to the nearest cubic foot.)

- (A) 1,150
- (B) 1,117
- (C) 167
- (D) 34



6. A waffle-flavored ice cream cone is pictured above. If the volume of the cone is  $14.4\pi$  cubic inches, what is the diameter of the cone (in inches)?
7. If the area of a square is doubled, how many times longer is a side of the larger square than is the length of the smaller square?
- (A) 4
  - (B) 2
  - (C)  $\sqrt{3}$
  - (D)  $\sqrt{2}$



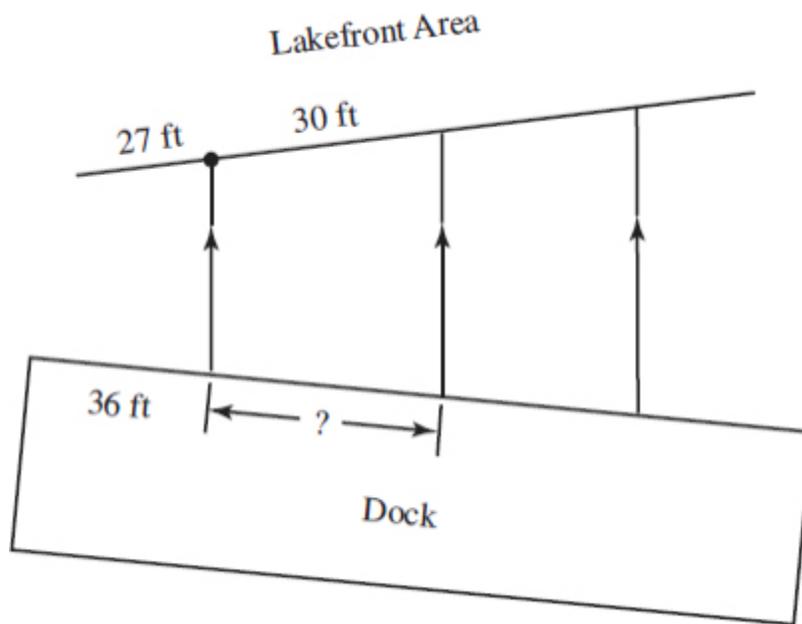
Height of Cylinder: 12'

Radius of Cylinder: 8'

Radius of Cone: 8'

Height of Cone: 8'

8. The diagram above represents a grain silo with a right cylindrical base and a top in the shape of a right cone. The owner believes he needs a new silo, which will store 10% more grain than his existing silo. Which of the following represents the volume of the new silo (in cubic feet)?
- (A) 2,123
  - (B) 2,678
  - (C) 2,948
  - (D) 3,243



9. The diagram above shows a lakefront area divided into small parcels for boat owners. Each area is marked by a rope to a dock where each owner secures his or her boat. The ropes between the parcels are parallel to one another. What is the length of the docking area in feet for the boat owner who has a 30-foot lakefront border illustrated above?
10. Oscar wants to place molding surrounding the floor in his study room that is in the shape of a rectangle. He remembers that the area of the room is 180 square feet and that the width of the room is 12 feet. However, he does not know the measure of the length of the room. His tape measure is out in the car, but Oscar thinks he can solve the problem with simple algebra. Assuming no waste, how many linear feet of floor molding should Oscar purchase?

### Answer Explanations

1. **(D)** Subtract the area of the circle from the area of the square.

$$\text{Area of a square} = \text{side} \times \text{side} = s^2$$

Divide the perimeter by 4 and square that value.

$$\left(\frac{64}{4}\right)^2 = 16^2 = 256$$

The area of the square is 256 square units.

$$\text{Area of a circle} = \pi r^2$$

The diameter of the circle is equal to the length of one of the sides of the square. Each side is 16, so the radius is 8 because the radius of a circle is one-half of the diameter.

$$\pi(8)^2 \approx 201.1$$

Subtract the area of the circle from the area of the square.

$$256 - 201.1 = 54.9$$

Divide the remaining area by 4 to find the shaded area.

$$54.9 \div 4 = 13.73$$

Divide the measure of the shaded area by the area of the square to find the probability of a coin landing in the shaded area.

$$13.73 \div 256 = 0.053$$

The diagram is assumed to be drawn to scale, so choice (A), 0.363, would appear to be much too large.

2. **(B)** The volume of the water in the first aquarium is 6 cubic feet because the volume, length  $\times$  width  $\times$  height, is  $3 \times 2 \times 1 = 6$ . The same volume of water is then poured into another aquarium that has dimensions 4 feet by 1 foot and that rises to a height of  $x$  feet. Calculate the height.

$$\begin{aligned}(4)(1)(x) &= 6 \\ 4x &= 6 \\ x &= 1.5\end{aligned}$$

3. **(B)** Find the radius of the sphere by using the formula  $V = \frac{4}{3}\pi r^3$ .

$$\begin{aligned}374 &= \frac{4}{3}(3.14)r^3 \\ 374 &= 4.19r^3 \\ 89.26 &= r^3 \\ 4.47 &= r\end{aligned}$$

The circumference of a circle is found by using the formula  $C = 2\pi r$ .

$$C = (2)(4.47)(3.14) = 28.07$$

Once you have calculated the length of the radius of the sphere, choice (A) can be eliminated since 4.47 is the radius of the great circle, not its circumference.

4. **(B)** Factoring  $16x^2 - 9y^2$  gives  $(4x + 3y)(4x - 3y)$ . The area of a rectangle is given by the formula Area = (length)(width) or  $A = lw$ . Since  $4x - 3y$  is the width,  $4x + 3y$  is the length. The perimeter of a rectangle is found by using the formula  $P = 2l + 2w$ .

$$2(4x + 3y) + 2(4x - 3y) = 8x + 6y + 8x - 6y = 16x$$

5. **(B)** Find the difference between the volume of the snowball at 10:00 A.M. and at 11:30 A.M. At 10:00 A.M., the radius of the snowball is 2 feet, so input that value into the volume formula of a sphere.

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(2)^3 \approx 33.5 \text{ ft}^3$$

Every 10 minutes, the radius of the snowball grows 6 inches, which is 0.5 feet. There are nine 10-minute intervals between 10:00 A.M. and 11:30 A.M., so multiply 9 by 0.5 to find how much the snowball's radius increased.

$$(9)(0.5) = 4.5$$

Add 4.5 feet to 2 feet to get a final radius of 6.5 feet. Replace  $r$  with 6.5 in the volume formula of a sphere.

$$V = \frac{4}{3}\pi(6.5)^3 \approx 1,150.3 \text{ ft}^3$$

Subtract 33.5 from 1,150.3 to find the increase in the snowball's volume.

$$1,150.3 - 33.5 \approx 1,117$$

6. **6** The volume of a cone is found by using the formula  $V = \frac{1}{3}\pi r^2 h$ , where  $r$  is the radius of the circular base and  $h$  is the height. Input the known data and solve for  $r$ .

$$14.4\pi = \frac{1}{3}\pi r^2(4.8)$$

$$14.4 = 1.6r^2$$

$$9 = r^2$$

$$3 = r$$

The question asks for the diameter, so multiply the radius, 3, by 2.

$$2 \times 3 = 6$$

7. **(D)** Imagine a square that has an area of 4 square units. Since the area of a square is found by using the formula,  $A = s^2$  (where  $s$  is the length of a side), then each side of the square is 2 because  $\sqrt{4} = 2$ . If the area of the larger square is double the area of the smaller square, the area of the larger square must be 8. Find the square root of 8 to find the length of a side.

$$\sqrt{8} = 2\sqrt{2}$$

When the area of a square is doubled, each side length increases by a factor of  $\sqrt{2}$ .

8. **(D)** Find the volume of the cylinder and the cone, and then add 10% to that sum.

The volume of a cylinder is  $\pi r^2 h$ .

$$\pi(8^2)(12) = 2,412$$

The volume of a cone is  $\frac{1}{3} \pi r^2 h$ .

$$\frac{1}{3}\pi(8^2)(8) = 536$$

Add the two quantities to find the volume of the silo.

$$2,412 + 536 = 2,948$$

The farmer needs 10% more space, so find 10% of 2,948 and add it to 2,948.

$$2,948 + (0.10)(2,948) = 3,243$$

Once the volume of the silo has been determined to be 2,948, you don't need to finish the calculation. Only choice (D), 3,243, is larger than 2,948.

9. **40** If three parallel lines intersect two transversals, they divide the transversals proportionally. The lakefront length that is 27 feet corresponds to a dock length of 36. The lakefront length that is 30 feet should have a docking length that is proportional to the 27:36 ratio. Set up a proportion to find the docking length of the wider dock. You will find it useful to simplify  $\frac{27}{36}$  to  $\frac{3}{4}$ .

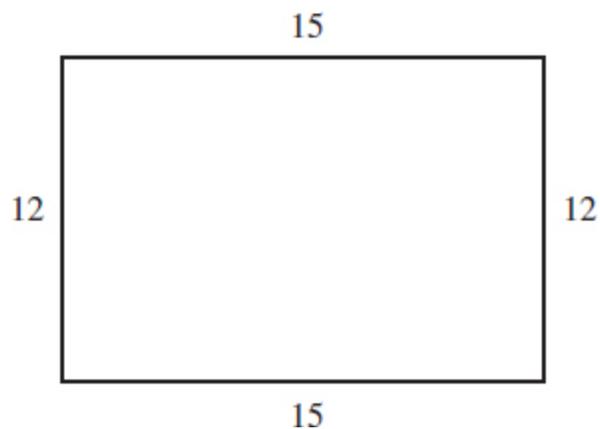
$$\frac{3}{4} = \frac{30}{x}$$
$$3x = 120$$
$$x = 40$$

10. **54** The molding, which surrounds the floor, is equal to the rectangle's perimeter. We are missing the length, but we know the area and width. The area of a rectangle can be found by using the formula Area = length  $\times$  width.

Input the known information to solve for  $l$ .

$$180 = l \times 12$$
$$15 = l$$

We now know the rectangular floor has the following dimensions:



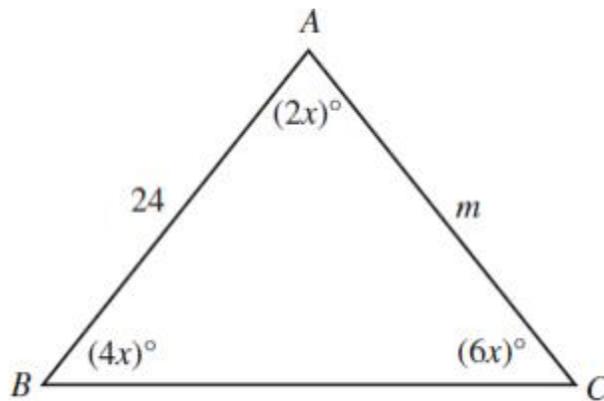
We can use the formula Perimeter =  $(2 \times l) + (2 \times w)$  to find the number of feet of molding Oscar needs to surround the floor.

$$(2 \times 15) + (2 \times 12) = 54$$

Oscar needs 54 feet of molding to surround the floor of his study.

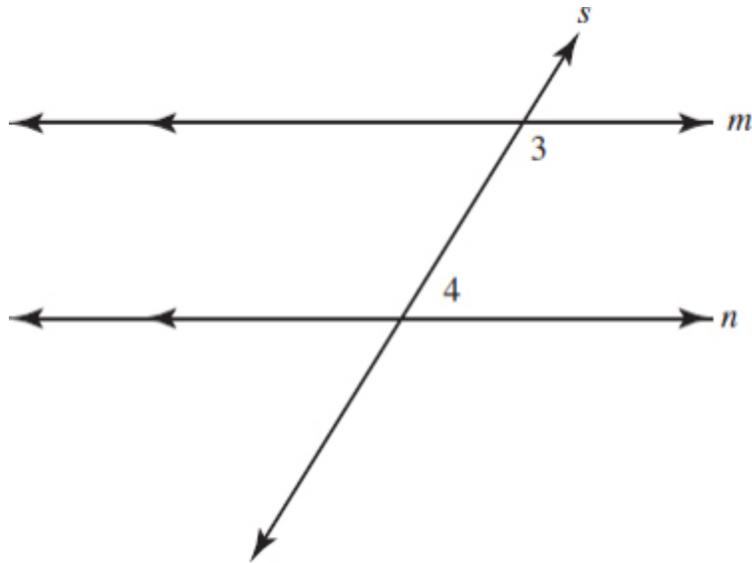
## Lines, Angles, and Triangles Drill

For multiple-choice questions, solve the problem and pick the correct answer from the provided choices. Each multiple-choice question has only one correct answer. For student-produced response questions, solve each problem and write down your answers separately.

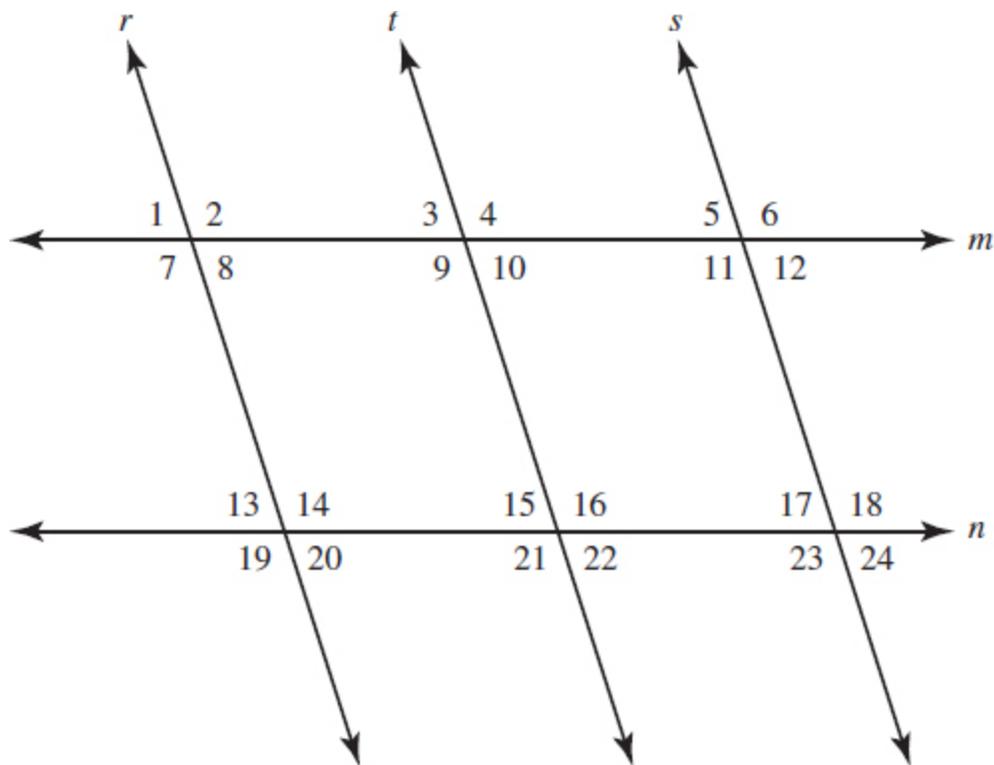


Note: Figure not drawn to scale

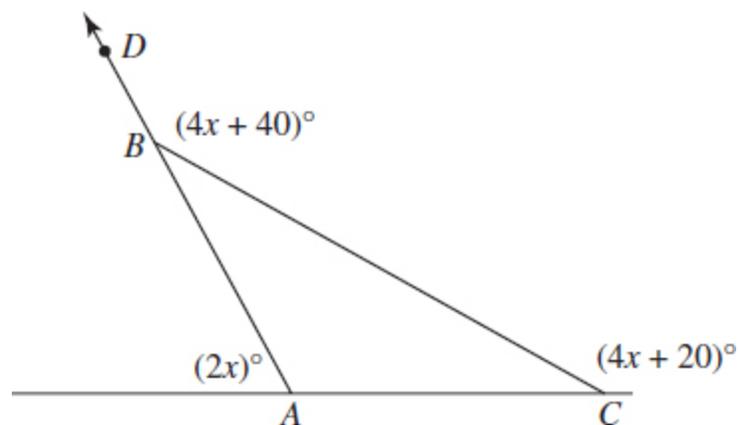
1. In  $\triangle ABC$ , what is the value of  $m$ ?
  - (A)  $12\sqrt{3}$
  - (B)  $12\sqrt{2}$
  - (C) 12
  - (D)  $3\sqrt{6}$



2. In the figure above, lines  $m$  and  $n$  are parallel. If  $m\angle 3 = (3x)^\circ$  and  $m\angle 4 = (2x)^\circ$ , what is  $m\angle 4$ ?
- (A)  $36^\circ$
  - (B)  $48^\circ$
  - (C)  $72^\circ$
  - (D)  $108^\circ$

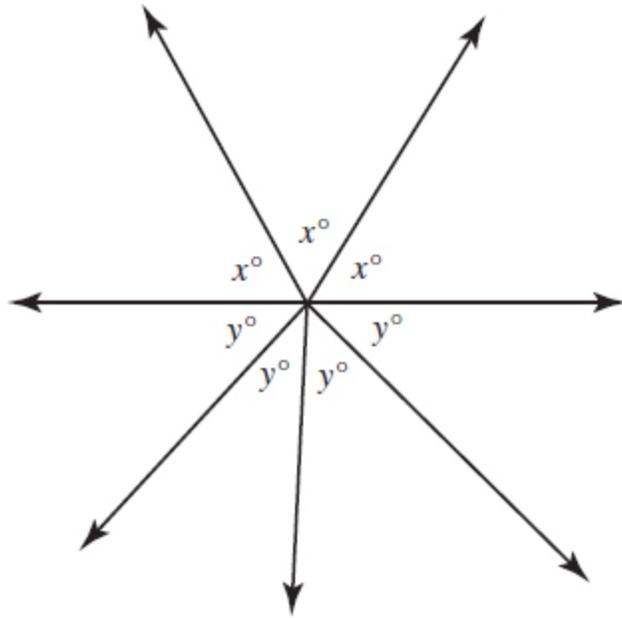


3. In the diagram above,  $r \parallel t \parallel s$  and  $m \parallel n$ . If the measure of  $\angle 3 = (2m + 18)^\circ$  and the measure of  $\angle 2 = (4m + 42)^\circ$ , what is the measure of  $\angle 18$ ?
- (A)  $122^\circ$
  - (B)  $61^\circ$
  - (C)  $58^\circ$
  - (D)  $20^\circ$



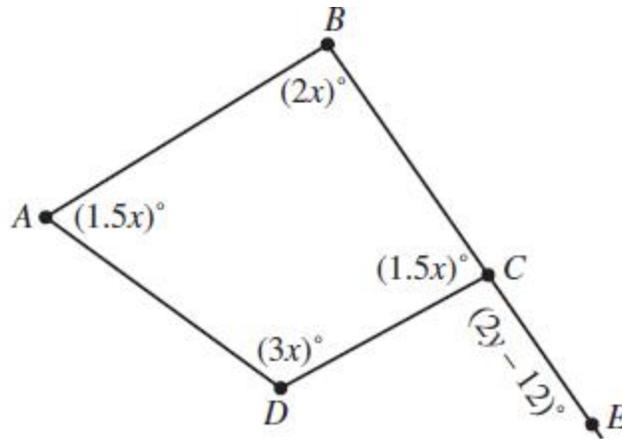
4. What is the measure of  $\angle ABC$ ?

- (A)  $20^\circ$
- (B)  $40^\circ$
- (C)  $65^\circ$
- (D)  $70^\circ$



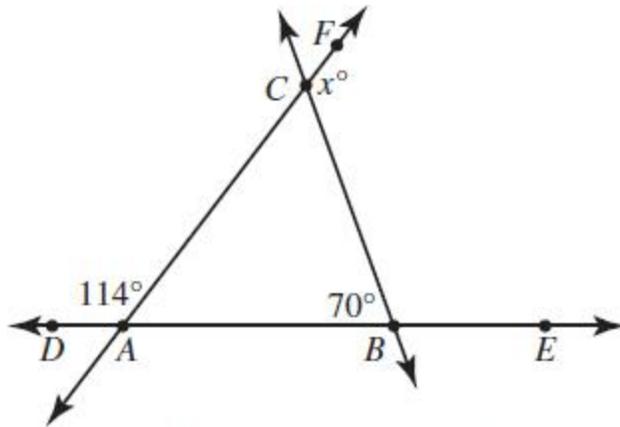
5. In the figure above, what is the value of  $(x + y) \div 2(x - y)$ ?

- (A) 3
- (B) 3.5
- (C) 4
- (D) 5.5



Note: Figure not drawn to scale

6. In the diagram above, what is the measure of  $y$ ?
  - (A)  $112.5^\circ$
  - (B)  $90^\circ$
  - (C)  $62.25^\circ$
  - (D)  $45^\circ$
  
7. Two sides of a triangle are 7 and 11 inches. If a third side of the triangle is also an integer, what is the product of the smallest and largest values that the side could be?
  
8. The sum of the measures of the interior angles of a polygon can be found by using the formula  $(n - 2)(180)$  where  $n$  is the number of angles. If the sum of the measures of a regular polygon is  $1,260^\circ$ , what is the degree measure of one interior angle? (Note: Ignore the degree symbol.)
  
9. The sum of the measures of the interior angles of a polygon can be found by using the formula  $(n - 2)(180)$  where  $n$  is the number of angles. A regular polygon has a side that measures 11 inches. If an exterior angle of the figure measures  $45^\circ$ , what is the measure in inches of the perimeter of the polygon?



Note: Figure not drawn to scale

10. In the diagram above, what is the value of  $x$ ? (Ignore the degree symbol.)

### Answer Explanations

1. **(A)** Find the measure of each angle by solving for  $x$ .

$$6x + 4x + 2x = 180$$

$$12x = 180$$

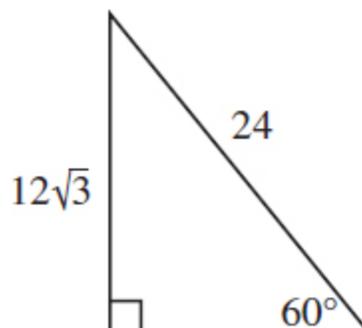
$$x = 15$$

$$2x = 30$$

$$4x = 60$$

$$6x = 90$$

$\triangle ABC$  is a 30-60-90 triangle. The hypotenuse measures 24, so  $m$  measures  $12\sqrt{3}$  because it is opposite the  $60^\circ$  angle.



2. **(C)**  $\angle 3$  and  $\angle 4$  are consecutive interior angles. When lines are parallel, consecutive interior angles are supplementary. Find their sum, and set that value equal to  $180^\circ$ .

$$3x + 2x = 180$$

$$5x = 180$$

$$x = 36$$

The measure of  $\angle 4$  is  $(2x)^\circ$ , so replace  $x$  with 36 and solve.

$$(2)(36) = 72$$

The figure is drawn to scale and angle 4 seems to be acute, so choice (D),  $108^\circ$ , can be eliminated at once.

3. **(A)** Both  $\angle 2$  and  $\angle 3$  are consecutive interior angles, so their sum is  $180^\circ$ . Solve for  $m$  by setting their sum equal to 180.

$$(2m + 18) + (4m + 42) = 180$$

$$6m + 60 = 180$$

$$6m = 120$$

$$m = 20$$

Substitute 20 for  $m$  and find the measure of  $\angle 2$ .

$$(4)(20) + 42 = 122$$

Since  $\angle 2$  and  $\angle 6$  are corresponding angles, their measures are equal. Since  $\angle 6$  and  $\angle 18$  are corresponding angles, their measures are also the same. Using the transitive property, we find that the measure of  $\angle 2$  equals the measure of  $\angle 6$ , which equals the measure of  $\angle 18$ , which all equal  $122^\circ$ . There are several alternate ways to solve this problem, but the easiest way is visually. The diagram is drawn to scale, and  $\angle 18$  appears to be obtuse. Solely choice (A),  $122^\circ$ , appears to have a measure greater than  $90^\circ$ .

4. **(A)** The sum of the measurements of the exterior angles of a polygon is  $360^\circ$ . Add the exterior angles of  $\triangle ABC$  and set them equal to 360 to find  $x$ .

$$\begin{aligned}(4x + 40) + (4x + 20) + 2x &= 360 \\ 10x + 60 &= 360 \\ 10x &= 300 \\ x &= 30\end{aligned}$$

$\angle DBC$  and  $\angle ABC$  are supplementary angles; their sum is  $180^\circ$ .

$$\angle DBC = 4x + 40 = 4(30) + 40 = 160$$

Therefore,  $m\angle ABC + 160 = 180$  and  $m\angle ABC = 20$ . The measure of  $\angle ABC$  is  $20^\circ$ .

5. **(B)** The sum of the angles above and below the horizontal line are each  $180^\circ$

$$\begin{aligned}3x &= 180 \\ x &= 60 \\ 4y &= 180 \\ y &= 45\end{aligned}$$

Inputting 60 and 45 for  $x$  and  $y$ , respectively, into the expression yields the following.

$$\begin{aligned}(x + y) \div 2(x - y) \\ (60 + 45) \div 2(60 - 45) \\ 105 \div 30 = 3.5\end{aligned}$$

6. **(C)** The sum of the measures of the angles in a quadrilateral is  $360^\circ$ . Solve for  $x$  by adding the angle measures and setting that value equal to 360.

$$1.5x + 3x + 1.5x + 2x = 360$$

$$8x = 360$$

$$x = 45$$

$m\angle BCD + m\angle ECD = 180$  because they form a linear pair.  $m\angle BCD$  equals 67.5 because  $1.5 \times 45 = 67.5$ . Therefore,  $2y - 12$  is 112.5 because it is supplementary to  $\angle BCD$ . Set  $2y - 12$  equal to 112.5 and solve for  $y$ .

$$112.5 = 2y - 12$$

$$124.5 = 2y$$

$$y = 62.25$$

The measure of  $y$  is  $62.25^\circ$ .

7. **85** The sum of two sides of a triangle is always larger than the length of the third side. Suppose that 11 is the measure of the longest side. Then  $7 + x > 11$  and  $x > 4$ . The smallest integer that satisfies this inequality is 5.

Suppose the largest side is unknown. Then  $7 + 11 > x$  and  $18 > x$ . The largest value that satisfies this inequality is 17.

The product of the smallest value, 5, and the largest value, 17, is  $5 \times 17 = 85$ .

8. **140** Use the formula  $(n - 2)(180)$  to find the number of angles included in the regular polygon.

$$(n - 2)(180) = 1,260$$

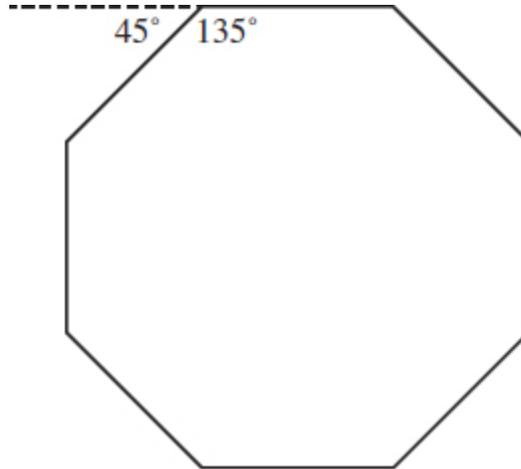
$$n - 2 = 7$$

$$n = 9$$

The regular polygon has 9 congruent interior angles. Divide 1,260 by 9 to find the measure of one of the angles.

$$1,260 \div 9 = 140$$

9. **88** An exterior angle and an interior angle of a polygon form a linear pair; the sum of their measures is  $180^\circ$ . Thus, one interior angle of the polygon measures  $135^\circ$  because  $180^\circ - 45^\circ = 135^\circ$ .



The formula  $(n - 2)(180)$  is used to calculate the sum of the measures of all of the interior angles of a polygon. Since  $n$  represents the number of congruent angles and congruent sides in a regular polygon, we can use  $\frac{(n - 2)(180)}{n}$  to find the measure of a single angle.

$$\frac{(n - 2)(180)}{n} = 135$$

Solve for  $n$ .

$$\begin{aligned}\frac{(n - 2)(180)}{n} &= 135 \\(n - 2)(180) &= 135n \\180n - 360 &= 135n \\-360 &= -45n \\n &= 8\end{aligned}$$

The polygon has 8 congruent sides and angles (an octagon). Since each side measures 11 inches, the perimeter is  $8 \times 11 = 88$  inches.

10. **136** Since  $m\angle DAC = 114^\circ$ , then  $m\angle CAB = 66^\circ$  because the two angles are supplementary. The sum of the measures of the angles in a triangle equals  $180^\circ$ . Therefore,  $m\angle ACB = 44^\circ$ .

$$66^\circ + 70^\circ + 44^\circ = 180^\circ$$

$\angle ACB$  and  $\angle FCB$  are supplementary angles, so their sum is  $180^\circ$ .

$$m\angle ACB + m\angle FCB = 180^\circ$$

Subtract  $44^\circ$  from  $180^\circ$  to find the value of  $x$ .

$$180^\circ - 44^\circ = 136^\circ$$

There is a theorem in geometry that states the measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles. We could have quickly found  $m\angle ACB$  by subtracting  $70^\circ$  from  $114^\circ$ .

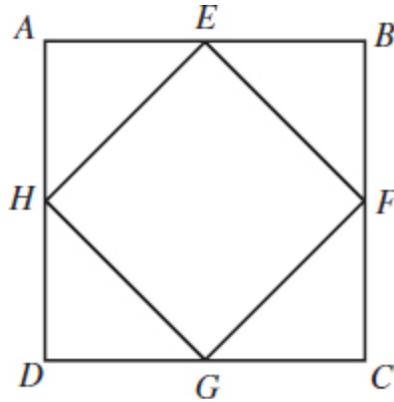
$$114^\circ - 70^\circ = 44^\circ$$

That step would have reduced your time solving for  $x$ .

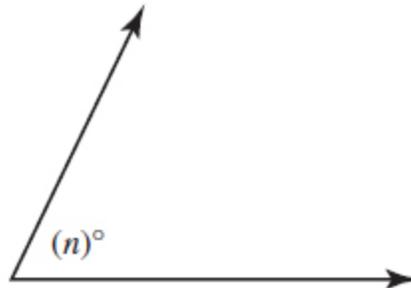
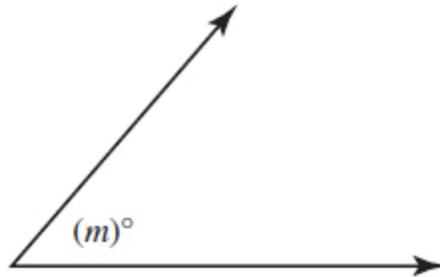
## Right Triangles and Trigonometry Drill

For multiple-choice questions, solve the problem and pick the correct answer from the provided choices. Each multiple-choice question has only one correct answer. For student-produced response questions, solve each problem and write down your answers separately.

1. In triangle  $ABC$ , the measure of angle  $B$  is  $90^\circ$ ,  $BC = 80$ , and  $AC = 100$ . Triangle  $DEF$  is like triangle  $ABC$  where vertices  $D$ ,  $E$ , and  $F$  correspond with vertices  $A$ ,  $B$ , and  $C$ , respectively. The scale factor of triangle  $DEF$  to triangle  $ABC$  is  $\frac{1}{10}$ . What is the value of  $\cos D$ ?



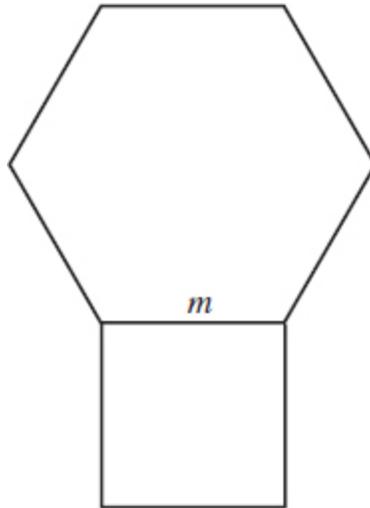
2. Square  $EHGF$  lies within square  $ABCD$  with  $E$ ,  $F$ ,  $G$ , and  $H$  all being midpoints. If the area of square  $EHGF$  is 64 units squared, what is the area of square  $ABCD$  (in square units)?
- (A) 36  
(B) 64  
(C) ~~96~~  
(D) 128
3. In a right triangle, one of the acute angles measures  $x^\circ$  where  $\sin x^\circ = \frac{5}{8}$ . What is the value of  $\cos(90^\circ - x^\circ)$ ?



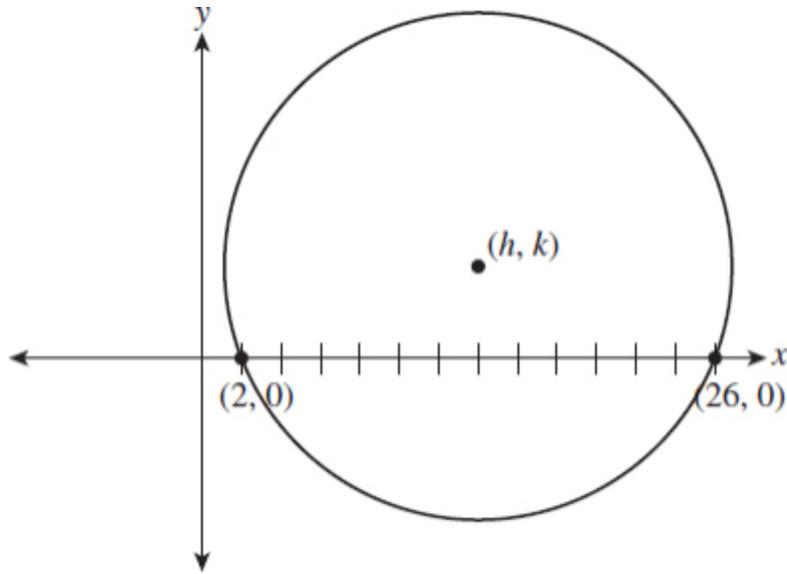
Note: Figures not drawn to scale

4. The angles shown above are complementary, and  $\sin(m^\circ) = \cos(n^\circ)$ . If  $m = (3x - 12)^\circ$  and  $n = (6x - 18)^\circ$ , what is the value of  $x$ ?

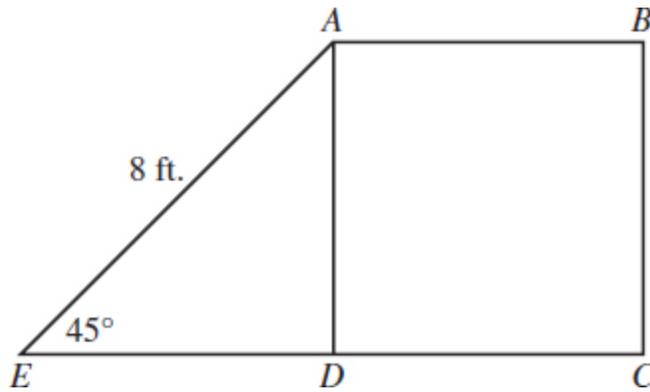
- (A) 13.3
- (B) 20.6
- (C) 30.25
- (D) 40.3



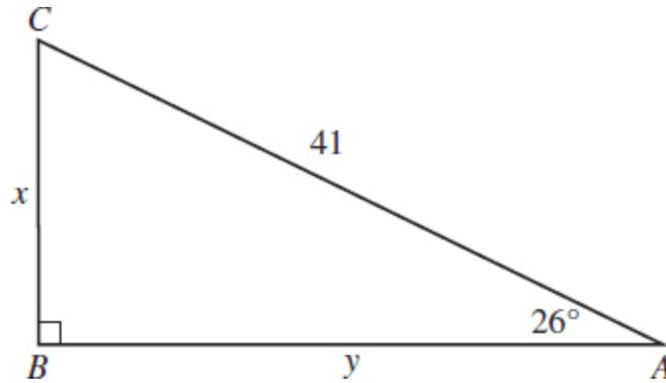
5. The figure above is a regular hexagon with sides of length  $m$  and a square with sides of length  $m$  as well. If the area of the hexagon is  $324\sqrt{3}$  inches, what is the area of the square in square inches?
- (A) 108
  - (B) 216
  - (C) 256
  - (D) 324



6. In the  $xy$ -plane, the circle shown above has center  $(h, k)$ . If the radius of the circle is 13, what is the value of  $k$ ?

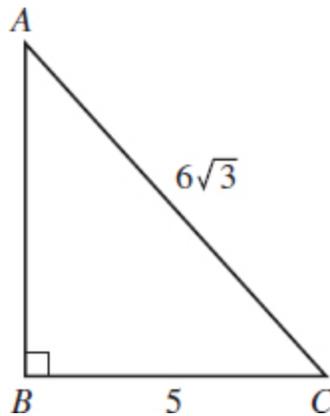


7. Square  $ABCD$  is adjacent to  $\triangle ADE$ . What is the area of square  $ABCD$ ? (Ignore square units in your answer.)



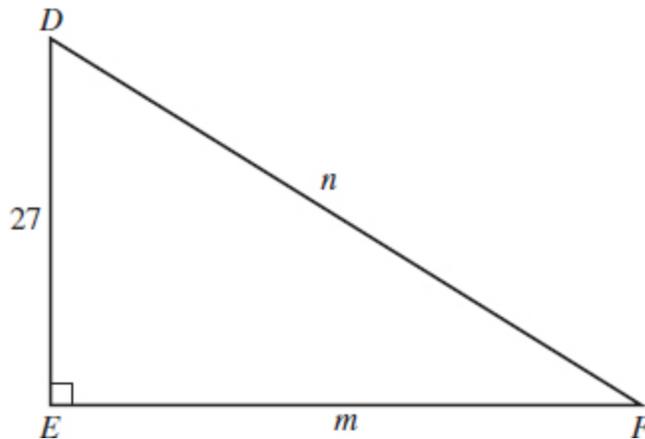
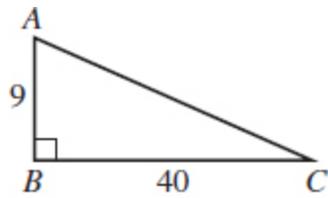
8. In  $\triangle CBA$  above, which equation will help find the measure of  $\overline{AB}$ ?

- (A)  $\tan 64^\circ = \frac{y}{41}$
- (B)  $\sin 64^\circ = \frac{y}{41}$
- (C)  $\tan 64^\circ = \frac{41}{y}$
- (D)  $\sin 26^\circ = \frac{x}{41}$



9. What is the perimeter of  $\triangle ABC$  shown above?

- (A)  $6\sqrt{3} + \sqrt{83}$
- (B)  $12 + 6\sqrt{3}$
- (C)  $5 + 6\sqrt{3} + \sqrt{83}$
- (D)  $3\sqrt{2} + 9\sqrt{3}$



10.  $\triangle ABC$  and  $\triangle DEF$  are similar triangles. What is the measure of  $DF$ ?

- (A) 123
- (B) 132
- (C) 144
- (D) 216

### Answer Explanations

1.  $\frac{3}{5}$  or .6 The scale factor of triangle  $DEF$  to triangle  $ABC$  is  $\frac{1}{10}$ . So multiply the length of the sides of triangle  $ABC$  by  $\frac{1}{10}$  to find the length of the sides of triangle  $DEF$ .

$$(100)\left(\frac{1}{10}\right) = 10 = DF$$

$$(80)\left(\frac{1}{10}\right) = 8 = EF$$

Use the Pythagorean theorem to calculate the length of  $DE$ .

$$\begin{aligned}
 a^2 + 8^2 &= 10^2 \\
 a^2 &= 36 \\
 a &= 6
 \end{aligned}$$

The cosine function is found by using the ratio  $\cos x = \frac{\text{adjacent side}}{\text{hypotenuse}}$ .  
 The side adjacent to angle  $D$  is 6 and the hypotenuse is 10.

$$\cos D = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{6}{10} = 0.6$$

2. **(D)** Square  $EFGH$  has an area that measures 64 square units, which means each side is 8 units. All four of the triangles are isosceles right triangles with hypotenuse 8. Using the formula for a 45-45-90 triangle, both legs of each right triangle are therefore  $4\sqrt{2}$  units. Each side of square  $ABCD$ , then, is  $8\sqrt{2}$  units long. The formula for the area of a square is  $\text{Area} = s^2$ , where  $s$  is the measure of a side.

$$\begin{aligned}
 \text{Area} &= s^2 \\
 \text{Area} &= (8\sqrt{2})^2 = 128
 \end{aligned}$$

The figures can be assumed to be to scale as there is no note indicating otherwise. We therefore can deduce square  $ABCD$  is larger than square  $EFGH$ . Thus, choice (A), 36, and choice (B), 64, can be immediately eliminated.

3.  $\frac{5}{8}$  or .625 By the complementary angle relationship for sine and cosine,  $\sin(x^\circ) = \cos(90^\circ - x^\circ)$ . Therefore,  $\cos(90^\circ - x^\circ) = \frac{5}{8}$ .

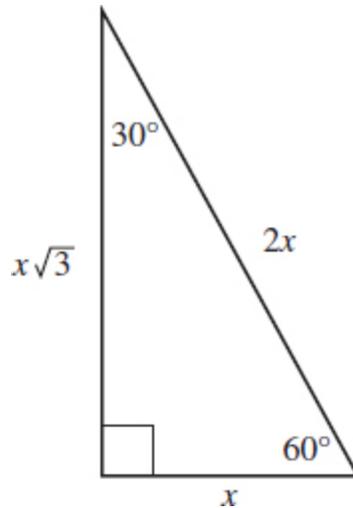
You can enter your answer as  $\frac{5}{8}$  or 0.625 (or just .625) into your digital test form.

4. **(A)** Since the angles are complementary and  $\sin(m^\circ) = \cos(n^\circ)$ , it follows from the complementary angle property of sines and cosines that  $m + n = 90$ . Substituting  $3x - 12$  for  $m$  and  $6x - 18$  for  $n$  gives  $(3x - 12) + (6x - 18) = 90$ , which simplifies to  $9x - 30 = 90$ . Therefore,  $9x = 120$ , and  $x = 13.\bar{3}$ .

5. **(B)** A regular hexagon can be divided into six equilateral triangles. Divide  $324\sqrt{3}$  by 6 to find the area of one of the equilateral triangles.

$$324\sqrt{3} \div 6 = 54\sqrt{3}$$

An equilateral triangle can be divided into two 30-60-90 triangles.



Multiply the base,  $x$ , by the height,  $\sqrt{3}$ , and by  $\frac{1}{2}$ . Then set that product equal to  $27\sqrt{3}$ , which is one-half of  $54\sqrt{3}$ .

$$\begin{aligned}\left(\frac{1}{2}\right)(x)(x\sqrt{3}) &= 27\sqrt{3} \\ \frac{1}{2}x^2 &= 27 \\ x^2 &= 54 \\ x &= 3\sqrt{6}\end{aligned}$$

Multiply  $3\sqrt{6}$  by 2 to find one side of the hexagon.

$$3\sqrt{6} \times 2 = 6\sqrt{6}$$

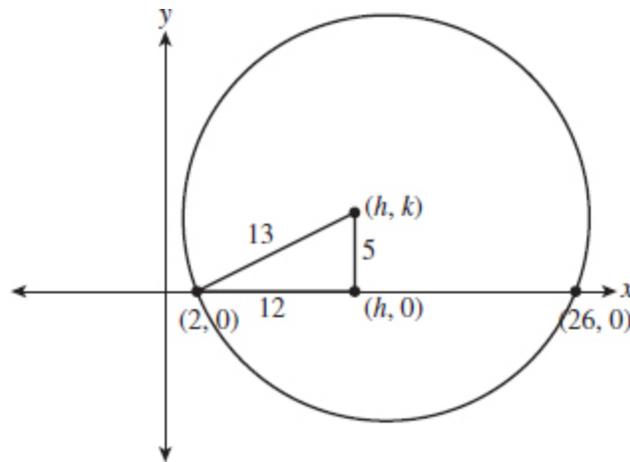
The side length of the hexagon is also the side length of the square. Square  $6\sqrt{6}$  to calculate the area of the square.

$$(6\sqrt{6})^2 = 216$$

6. **5** The length of the segment connecting  $(2, 0)$  and  $(26, 0)$  is 24 because  $26 - 2 = 24$ . Extend  $(h, k)$  to this segment to create an altitude. From  $(h, 0)$  to  $(2, 0)$  is 12 units because a segment drawn from the center and perpendicular to a segment bisects the segment. Given that the radius of the circle is 13, use the Pythagorean theorem to calculate the height,  $k$ , of the circle.

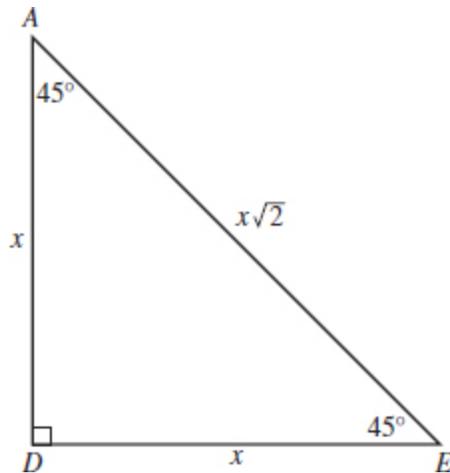
$$\begin{aligned}a^2 + 12^2 &= 13^2 \\a^2 + 144 &= 169 \\a^2 &= 25 \\a &= 5\end{aligned}$$

The reader may realize that the triangle is the Pythagorean triple 5-12-13.



The value of  $k$  is 5.

7. **32** A square has four right angles, so the measure of  $\angle ADE$  is  $90^\circ$ . Therefore,  $\triangle ADE$  is a 45-45-90 triangle.



Set  $8$  equal to  $x\sqrt{2}$  and solve for  $x$ .

$$\begin{aligned}
 x\sqrt{2} &= 8 \\
 x &= \frac{8}{\sqrt{2}} \\
 x &= \frac{8}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\
 x &= \frac{8\sqrt{2}}{\sqrt{2}} = 4\sqrt{2}
 \end{aligned}$$

Each side of the square is  $4\sqrt{2}$ . The formula for the area of a square is  $A = s^2$ , where  $s$  represents the length of one side. Replace  $s$  with  $4\sqrt{2}$  and solve.

$$\begin{aligned}
 A &= (4\sqrt{2})^2 \\
 A &= 32
 \end{aligned}$$

8. **(B)** The acute angles of a right triangle are complementary, so  $m\angle C = 64^\circ$ . The length of  $\overline{BA}$ ,  $y$ , is opposite  $\angle C$ . So use the formula  $\sin = \frac{\text{opposite side}}{\text{hypotenuse}}$  to find the length of  $\overline{BA}$ .

$$\sin 64^\circ = \frac{y}{41}$$

This equation reflects choice (B).

9. **(C)** Use the Pythagorean theorem to calculate the length of  $\overline{AB}$ .

$$a^2 + 5^2 = (6\sqrt{3})^2$$

$$a^2 + 25 = 108$$

$$a^2 = 83$$

$$a = \sqrt{83}$$

Add the measures of the sides of  $\triangle ABC$ .

$$5 + \sqrt{83} + 6\sqrt{3}$$

None of the quantities above are like terms, so  $5 + \sqrt{83} + 6\sqrt{3}$  is the perimeter.

10. **(A)** Find the value of  $\overline{AC}$  by using the Pythagorean theorem.

$$9^2 + 40^2 = c^2$$

$$1,681 = c^2$$

$$41 = c$$

The two triangles are similar, so the sides are proportional.

$$\frac{AB}{AC} = \frac{DE}{DF}$$

$$\frac{9}{41} = \frac{27}{x}$$

Cross-multiply and solve for  $x$ .

$$(9)(x) = (27)(41)$$

$$9x = 1,107$$

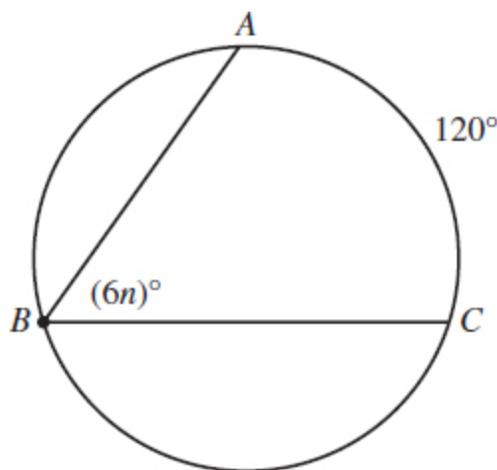
$$x = 123$$

The measure of  $\overline{DF}$  is 123 units.

## Circles Drill

For multiple-choice questions, solve the problem and pick the correct answer from the provided choices. Each multiple-choice question has only one correct answer. For student-produced response questions, solve each problem and write down your answers separately.

1. What is the area of a circle that has a circumference that measures  $38\pi$ ?  
(A)  $144\pi$   
(B)  $225\pi$   
(C)  $361\pi$   
(D)  $400\pi$
2. What are the coordinates of the center of a circle with the standard form  $x^2 + y^2 - 2x + 4y = 4$ ?  
(A)  $(1, -2)$   
(B)  $(-1, 2)$   
(C)  $(4, 1)$   
(D)  $(-1, 4)$



3. What is the value of  $n$  in the diagram above?

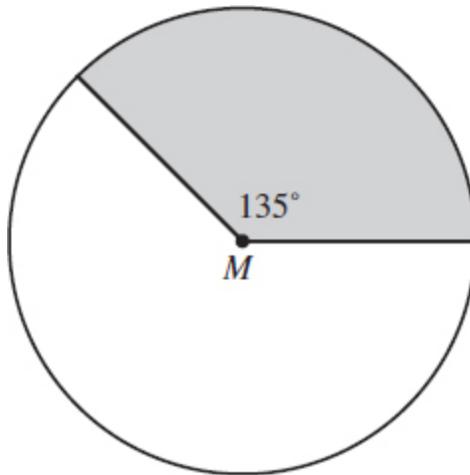
- (A) 5
- (B) 10
- (C) 20
- (D) No conclusion can be drawn from the diagram.

$$x^2 + y^2 + 6x - 8y = 12$$

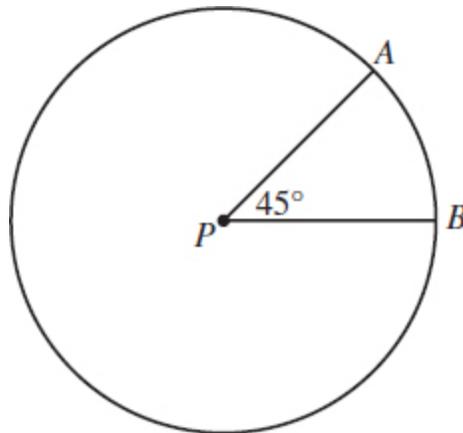
4. The equation of a circle in the  $xy$ -plane is provided above. What is the radius of the circle?
- (A) 5
  - (B)  $3\sqrt{3}$
  - (C) 6
  - (D)  $\sqrt{37}$
5. In a circle with center  $P$ , central angle  $MPN$  has a measure of  $\frac{3\pi}{4}$  radians. The area of the sector formed by the central angle  $MPN$  is what fraction of the circle?
6. In the  $xy$ -plane, the graph of a circle is  $2x^2 + 8x + 2y^2 + 12y = 36$ . What is the diameter of the circle?
- (A)  $\sqrt{31}$
  - (B)  $\sqrt{62}$
  - (C)  $2\sqrt{31}$
  - (D)  $2\sqrt{62}$

$$x^2 + y^2 - 12x + 3y = -7$$

7. The equation above models a circle in the  $xy$ -plane. What is the measure of the diameter of the circle?
- (A)  $\frac{5\sqrt{5}}{2}$
  - (B)  $5\sqrt{5}$
  - (C)  $10\sqrt{5}$
  - (D)  $10\sqrt{10}$



8. Point  $M$  is the center of the circle shown above. The shaded area is what fraction of the area of the circle?
9. A central angle in a circle has a measure of  $34.4^\circ$ , and the measure of its sector is 139.6 square units. To the nearest tenth of a unit, what is the measure of the circle's radius? (Use  $\pi = 3.14$ )



10. Circle  $P$  has a central angle that measures  $45^\circ$  and a circumference that measures 88 inches. What is the measure of  $\widehat{AB}$ ?
  - (A) 44 inches
  - (B) 22 inches
  - (C) 11 inches
  - (D) 5.5 inches

## Answer Explanations

1. **(C)** Find the radius of a circle with a circumference measuring  $38\pi$ .

$$\begin{aligned}2\pi r &= 38\pi \\ r &= 19\end{aligned}$$

Substitute  $r = 19$  in the area formula of a circle.

$$\begin{aligned}A &= \pi r^2 \\ A &= \pi(19)^2 = 361\pi\end{aligned}$$

2. **(A)** Transform the equation to graphing form by using the equation  $(x - h)^2 + (y - k)^2 = r^2$  where  $(h, k)$  is the center and  $r$  is the radius. Use completing the square method to transform the equation.

$$x^2 - 2x + y^2 + 4y = 4$$

Divide both  $-2$  (the coefficient of  $x$ ) and  $4$  (the coefficient of  $y$ ) by  $2$ , and square each result.

$$\begin{aligned}\frac{-2}{2} &= -1 \\ (-1)^2 &= 1 \\ \frac{4}{2} &= 2 \\ (2)^2 &= 4\end{aligned}$$

Add  $-1$  and  $4$  to both sides of the equation.

$$x^2 - 2x + 1 + y^2 + 4y + 4 = 4 + 1 + 4$$

Factor the trinomials, and simplify the left side of the equation.

$$(x - 1)^2 + (y + 2)^2 = 9$$

By inspection, the center is  $(1, -2)$ . There will be questions that ask for the radius, which is 3 in this question.

3. **(B)**  $\angle ABC$  is an inscribed angle because its vertex is on the circle. An inscribed angle measures one-half of the measure of the intercepted arc.

$$6n = \frac{1}{2}(120)$$

$$6n = 60$$

$$n = 10$$

4. **(D)** The graphing form of a circle is  $y = (x - h)^2 + (y - k)^2 = r^2$ , where  $(h, k)$  is the center of the circle and  $r$  is the radius. Convert the equation to graphing form by completing the square.

$$\begin{aligned}x^2 + y^2 + 6x - 8y &= 12 \\x^2 + 6x + 9 + y^2 - 8y + 16 &= 12 + 9 + 16 \\(x + 3)^2 + (y - 4)^2 &= 37\end{aligned}$$

The radius squared is 37, so find its square root to calculate the length of the radius.

$$r^2 = 37$$

$$r = \sqrt{37}$$

5.  $\frac{3}{8}$  or .375 The area of a circle is surrounded by an angle of measure  $2\pi$  radians. Find the ratio of the central angle,  $\frac{3\pi}{4}$ , to the entire circle.

$$\frac{\frac{3\pi}{4}}{\frac{2\pi}{1}} = \frac{3\pi}{4} \times \frac{1}{2\pi} = \frac{3}{8} \text{ or } .375$$

6. **(C)** The graphing form of a circle is  $(x - h)^2 + (y - k)^2 = r^2$  where  $(h, k)$  is the center of the circle and  $r$  is the radius of the circle.

Divide by 2 to simplify the equation.

$$(2x^2 + 8x + 2y^2 + 12y) \div 2 = 36 \div 2$$

$$x^2 + 4x + y^2 + 6y = 18$$

Next, complete the square for both the  $x$  and  $y$  components of the equation.

$$x^2 + 4x + 4 + y^2 + 6y + 9 = 18 + 4 + 9$$

Factor the left side of the equation and simplify the right side.

$$(x + 2)^2 + (y + 3)^2 = 31$$

In this circle,  $r^2 = 31$ . So  $r = \sqrt{31}$ . The question requires the diameter of the circle, so multiply the radius by 2.

$$2 \times \sqrt{31} = 2\sqrt{31}$$

7. **(B)** Calculate the length of the diameter by first finding the radius. Use completing the square to transform the equation from standard form to graphing form.

$$x^2 + y^2 - 12x + 3y = -7$$

$$x^2 - 12x + y^2 + 3y = -7$$

$$x^2 - 12x + 36 + y^2 + 3y + \frac{9}{4} = -7 + 36 + \frac{9}{4}$$

$$(x - 6)^2 + \left(y + \frac{3}{2}\right)^2 = \frac{125}{4}$$

The graphing form of a circle is  $(x - h)^2 + (y - k)^2 = r^2$  in which  $(h, k)$  are the coordinates of the center and  $r$  is the length of the radius. Find  $r$ .

$$r^2 = \frac{125}{4}$$

$$r = \frac{5\sqrt{5}}{2}$$

Multiply the radius by 2 to find the measure of the diameter.

$$d = (2)\left(\frac{5\sqrt{5}}{2}\right) = 5\sqrt{5}$$

8.  $\frac{3}{8}$  or .375 The shaded area has a central angle that measures  $135^\circ$ . Therefore, the shaded area is  $\frac{135}{360}$  of the circle. Reduce  $\frac{135}{360}$  to  $\frac{3}{8}$  or express it as the decimal .375.
9. **21.6** The area of a sector is found by using the following formula.

$$A = \frac{m}{360}\pi r^2$$

where  $m$  = measure of the central angle and  $r$  = length of the radius.  
Input the known data and solve for  $r$ .

$$\begin{aligned} 139.6 &= \frac{34.4}{360}\pi r^2 \\ 465.33 &\approx r^2 \\ 21.6 &\approx r \end{aligned}$$

10. **(C)** Find the length of  $\widehat{AB}$  by using the formula  $\left(\frac{m}{360}\right)$  (circumference) in which  $m$  is the measure of the central angle.

$$\left(\frac{45}{360}\right)(88) = 11$$

The length of  $\widehat{AB}$  is 11 inches.

## Levels of Difficulty

When taking the digital SAT, test takers will notice that the level of difficulty increases the further they progress through the test. This final section simulates that aspect of the exam. The reader will now practice questions grouped by levels of difficulty: easy/medium/difficult.

### Questions by Level of Difficulty

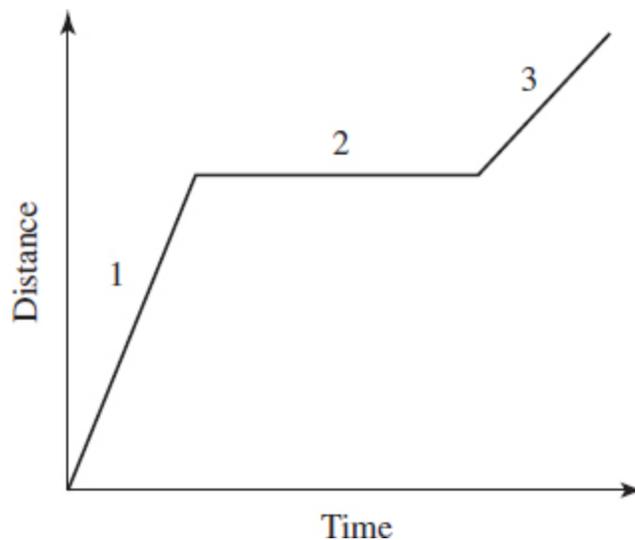
#### Easy Questions

For multiple-choice questions, solve the problem and pick the correct answer from the provided choices. Each multiple-choice question has only one correct answer. For student-produced response questions, solve each problem and write down your answers separately.

1. What is the value of  $x$  in the equation  $7x + 4 = 95$ ?
  - (A) 12
  - (B) 13
  - (C) 14
  - (D) 15
2. What number is 84% of 17?
3. If  $2x - 6 = 18$ , what is the value of  $x - 3$ ?
4. If  $\frac{r-2}{6} = n$  and  $n = 5$ , what is the value of  $r$ ?
  - (A) 28
  - (B) 30
  - (C) 32
  - (D) 42
5. Shelley is a salesperson for a local furniture store. She receives a \$24.00 commission for every recliner she sells and \$63.00 for every dinette set. If Shelley sells  $r$  dinette sets and  $d$  recliners during a work shift, which expression represents her pay for that day?
  - (A)  $63r + 24d$
  - (B)  $24r + 63d$
  - (C)  $24r - 63d$
  - (D)  $63r - 24d$
6. Cindy owns a candy store and sells individual candies as well as mixes. Her most popular mix is the chocolate and mint collection. The

ratio of chocolates to mints in this candy mix is 3 to 5. If she is mixing 7 pounds of the mix, how many pounds are chocolates?

7. Warren has a streaming video account with an online movie channel. The cost for each movie he rents is \$2.25, and his monthly membership fee is \$4.00. If his invoice for the past month was \$24.25, how many movies did Warren rent?
- (A) 6  
(B) 7  
(C) 8  
(D) 9
8. The product of an integer,  $n$ , and 5, increased by 12, is  $-13$ . What is the value of  $n$ ?
- (A) 25  
(B) 5  
(C)  $\frac{1}{5}$   
(D)  $-5$
9. Kareem has purchased a hybrid automobile that has an average fuel efficiency of 40 miles per gallon. He filled his 12-gallon tank yesterday, but it now registers  $\frac{3}{4}$  full. How many miles did Kareem drive if his mileage reflected his average fuel efficiency?



10. Thomas rides his bike to the park and then sits down on a bench to rest. After his rest, he jogs several miles. Which part(s) of the graph represent(s) the time he spent sitting on the bench?
- (A) 1 and 2
  - (B) 2 only
  - (C) 2 and 3
  - (D) 3 only

### Answer Explanations

1. **(B)** Solve for  $x$ .

$$\begin{aligned}7x + 4 &= 95 \\7x + 4 - 4 &= 95 - 4 \\7x &= 91 \\\frac{7x}{7} &= \frac{91}{7} \\x &= 13\end{aligned}$$

2. **14.28** Set up an equation to solve the problem.

$$\begin{aligned}x &= (0.84)(17) \\x &= 14.28\end{aligned}$$

3. **9** Solve for  $x$ .

$$\begin{aligned}2x - 6 &= 18 \\2x &= 24 \\x &= 12\end{aligned}$$

Substitute  $x = 12$  into  $x - 3$ .

$$x - 3 = 9$$

There is a faster way to solve this problem. Divide  $2x - 6 = 18$  by 2.

$$(2x - 6) \div 2 = 18 \div 2$$
$$x - 3 = 9$$

4. **(C)** Substitute 5 for  $n$  as provided in the original equation.

$$\frac{r - 2}{6} = 5$$

Next, multiply both sides of the equation by 6 to isolate  $r - 2$ .

$$(6)\left(\frac{r - 2}{6}\right) = 5(6)$$
$$r - 2 = 30$$

Add 2 to both sides of the equation.

$$r - 2 + 2 = 30 + 2$$
$$r = 32$$

5. **(A)** If Shelley sells 10 recliners and 10 dinette sets, her commission would be  $10(24) + 10(63) = 870$ . In other words, multiply each commission by the number of units she sells of a particular item. If she sells  $r$  dinette sets and  $d$  recliners, her commission would be  $63r + 24d$ .
6.  **$\frac{21}{8}$  or 2.62 or 2.63** Since the ratio of chocolates to mints is 3 to 5, the ratio of chocolates to the mix is  $\frac{3}{3+5} = \frac{3}{8}$ . Remember, there are 8 parts to the mix: 3 parts chocolate and 5 parts mints. Multiply 7 pounds by  $\frac{3}{8}$  to find the number of pounds of chocolates in a 7-pound mix.

$$\frac{3}{8} \times 7 = \frac{21}{8} \text{ or } 2.62 \text{ or } 2.63$$

7. **(D)** Let  $x$  equal the number of movies rented by Warren in the past month.

$$2.25x + 4 = 24.25$$
$$2.25x = 20.25$$

Divide both sides of the equation by 2.25.

$$\frac{2.25x}{2.25} = \frac{20.25}{2.25}$$
$$x = 9$$

8. **(D)** The product of two values is the result of multiplying them. “Increased by” means use addition. We therefore arrive at the following equation.

$$5n + 12 = -13$$

Begin isolating the variable by subtracting 12 from both sides of the equation.

$$5n + 12 - 12 = -13 - 12$$
$$5n = -25$$

Complete the problem by dividing both sides of the equation by 5.

$$\frac{5n}{5} = \frac{-25}{5}$$
$$n = -5$$

Check your solution by replacing  $n$  with  $-5$  in the original equation.

$$5(-5) + 12 = -13$$
$$-25 + 12 = -13$$
$$-13 = -13$$

9. **120** Kareem’s gas tank registered  $\frac{3}{4}$  full, which means he has used  $\frac{1}{4}$  of his full tank, which has a capacity of 12 gallons. Multiply  $\frac{1}{4}$  by 12 to find how many gallons Kareem used.

$$\frac{1}{4} \times 12 = 3$$

Kareem used 3 gallons of gas. Each gallon lets him drive 40 miles, so he drove a total of  $3 \times 40 = 120$  miles.

10. **(B)** Part 2 of the graph is flat, indicating that time passed but no additional distance was covered. Parts 1 and 3 show that both time and distance were covered as evidenced by the upward behavior of the graph.

## Medium Questions

For multiple-choice questions, solve the problem and pick the correct answer from the provided choices. Each multiple-choice question has only one correct answer. For student-produced response questions, solve each problem and write down your answers separately.

$$\begin{aligned}\frac{2}{3}x - \frac{5}{9}y &= -7 \\ bx - \frac{10}{3}y &= 17\end{aligned}$$

1. In the system of equations above,  $b$  is a constant. If the system has no solution, what is the value of  $b$ ?
- (A)  $-3.75$   
(B)  $3.75$   
(C)  $4$   
(D)  $4.4$

Day	Thursday	Friday	Saturday
Chance of rain	65%	74%	31%

2. The coach of the Denver High School baseball team will have to cancel an upcoming tournament if the weather conditions forecast

rain. The table presents the probability of rain for the next three days. If it doesn't rain on Friday or Saturday, the coach can plan on holding the tournament at his high school. What is the probability that it will rain on Thursday but not Friday or Saturday?

- (A) 18.7%
- (B) 16.3%
- (C) 14.9%
- (D) 11.7%

3. If  $2x + 3y = 17$  and  $5x - 7y = 31$ , what is the value of  $\frac{7x - 4y}{8}$ ?

- (A) 6
- (B) 7
- (C) 8
- (D) 12

$$ax + by = 8$$
$$4x + 6y = 48$$

4. The system of equations above has infinite solutions. If  $a$  and  $b$  are constants, what is the value of  $\frac{b}{a}$ ?

$x$	1	2	3
$g(x)$	3	5	7

$x$	3	4	5
$h(x)$	8	16	24

5. Values for  $g(x)$  and  $h(x)$  are listed above. What is the value for  $h(g(2))$ ?

- (A) 24
- (B) 36
- (C) 48
- (D) 72

6. Students at a local art school wish to create a Styrofoam pyramid. The pyramid will have a rectangular base that measures 6 feet by 8 feet and will have a height that measures 12 feet. If each cubic foot weighs 4 ounces and the cost per pound of Styrofoam is \$0.50 per pound, what will be the cost to create the pyramid? (Note: 16 ounces = 1 pound) (Ignore the \$ sign.)
7. A circle in the  $xy$ -plane has the equation  $(x + 2)^2 + (y - 3)^2 = 25$ . Which of the following coordinate pairs lies in the interior of the circle?
- (A)  $(-3, 2)$   
 (B)  $(7, -4)$   
 (C)  $(3, -6)$   
 (D)  $(0, -11)$
8. The expression  $\frac{4n + 6}{n - 4}$  is equivalent to which of the following?

- (A)  $-\frac{4 + 6}{4}$   
 (B)  $4 - \frac{6}{4}$   
 (C)  $4 + \frac{6}{n - 4}$   
 (D)  $4 + \frac{22}{n - 4}$

$$g(x) = (x + 4)(x - 8)$$

9. Which of the following shows an equivalent form of the function  $g$  so that a minimum value of  $g(x)$  appears as a constant or coefficient?
- (A)  $g(x) = x^2 - 32$   
 (B)  $g(x) = x^2 - 4x - 32$   
 (C)  $g(x) = (x - 2)^2 - 36$   
 (D)  $g(x) = (x - 4)^2 - 32$
10. The distance between the points  $(7, 5)$  and  $(x, 4)$  is 8. What is/are the coordinate(s) of  $x$ ?
- (A)  $7 + 3\sqrt{7}$

$$(B) 7 - 3\sqrt{7}$$

$$(C) 3 \pm \sqrt{7}$$

$$(D) 7 \pm 3\sqrt{7}$$

## Answer Explanations

1. **(C)** If the system of equations has no solution, each equation has the same slope and a different  $y$ -intercept, thus making the lines parallel. In this system of equations, the  $x$ 's and  $y$ 's must be multiples of one another.

Find the multiple by dividing  $-\frac{10}{3}y$  by  $-\frac{5}{9}y$ .

$$-\frac{10}{3}y \div -\frac{5}{9}y = 6$$

Multiply  $\frac{2}{3}$  by 6 to find the value of  $b$ .

$$\frac{2}{3} \times 6 = 4$$

Make sure the constants are different to ensure the lines are parallel. Since  $-7(6)$  equals 42 and does not equal 17, the constants are different.

2. **(D)** The probability of rain on Thursday is 65%. Since the probability of rain on Friday and Saturday are 74% and 31%, respectively, subtract each from 100% to find the probability of no rain on both days.

No rain on Friday:  $100\% - 74\% = 26\%$

No rain on Saturday:  $100\% - 31\% = 69\%$

Each day's forecast is independent of the others, so multiply the three probabilities.

$$(0.65)(0.26)(0.69) = 0.1166 \approx 11.7\%$$

3. **(A)** Although this system of equations can be solved via the substitution or elimination method, it is much easier to add the two equations together.

$$\begin{array}{r} 2x + 3y = 17 \\ + 5x - 7y = 31 \\ \hline 7x - 4y = 48 \\ \frac{7x - 4y}{8} = \frac{48}{8} = 6 \end{array}$$

4.  $\frac{3}{2}$  or 1.5 If the system of equations has infinite solutions, they are equations that represent the same line. Multiply the top equation by 6 to ensure that the constant terms are the same in both equations.

$$6(ax + by = 8) = 6ax + 6by = 48$$

The system of equations now appears as follows:

$$\begin{array}{r} 6ax + 6by = 48 \\ 4x + 6y = 48 \end{array}$$

We can now conclude that  $6ax = 4x$  and  $6by = 6y$ .

$$\begin{array}{r} \frac{6ax}{6x} = \frac{4x}{6x} \\ a = \frac{2}{3} \\ \frac{6by}{6y} = \frac{6y}{6y} \\ b = 1 \end{array}$$

Now find  $\frac{b}{a}$ .

$$\frac{b}{a} = \frac{1}{\frac{2}{3}} = \frac{3}{2}$$

The correct answer can also be entered as 1.5.

5. **(A)** Using the values chart, note that when  $x = 2$ ,  $g(2) = 5$ . Now check the  $h(x)$  values chart for  $x = 5$ . We find  $h(5) = 24$ . Therefore,  $h(g(2)) = 24$ .
6. **24.0 or 24** Find the volume of the pyramid using the formula  $V = \frac{1}{3}Bh$ , where  $B$  is the base area of the pyramid and  $h$  is its height.

Use the formula Area = (length)(width) to find the base area.

$$\text{Area} = 6 \times 8 = 48 \text{ square feet}$$

Replace  $B$  with 48 in the volume formula.

$$V = \left(\frac{1}{3}\right)(48)(12) = 192$$

The pyramid has a volume that measures 192 cubic feet. Since each cubic foot weighs 4 ounces, multiply 192 by 4 to find its weight in ounces.

$$192 \times 4 = 768$$

Divide 768 by 16, the number of ounces in a pound, to find the weight of the pyramid in pounds.

$$768 \div 16 = 48$$

Multiply 48 by \$0.50 to find the cost for the Styrofoam to make the pyramid.

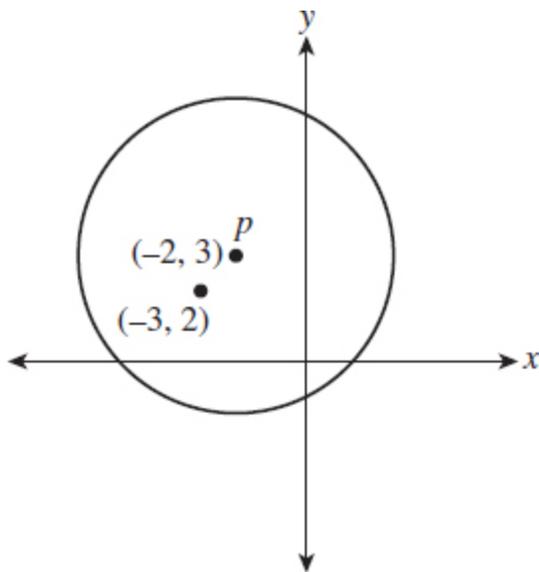
$$48 \times \$0.50 = \$24.00$$

7. **(A)** In order for a point to lie in the interior of  $(x + 2)^2 + (y - 3)^2 = 25$ , its coordinates must satisfy  $(x + 2)^2 + (y - 3)^2 < 25$ . Substitute choice (A),  $(-3, 2)$ , into the inequality.

$$(-3 + 2)^2 + (2 - 3)^2 < 25$$

$$2 < 25$$

An alternate way to answer the question is by sketching a graph.



8. **(D)** Use long division to assess the equivalent statement.

$$\begin{array}{r}
 4 + \frac{22}{n-4} \\
 n-4 \overline{) 4n+6} \\
 \underline{4n-16} \\
 22
 \end{array}$$

$4 + \frac{22}{n-4}$  is equivalent to  $\frac{4n+6}{n-4}$ .

9. **(C)** The vertex, or graphing, form of a parabola shows the vertex. Because the leading coefficient of  $g(x) = (x + 4)(x - 8)$  is positive, the parabola opens up and its vertex is a minimum.

Follow these steps to transform the equation from factored form to vertex form.

$$g(x) = (x + 4)(x - 8)$$

$$g(x) = x^2 - 4x - 32$$

$$g(x) = x^2 - 4x - 32$$

$$g(x) = x^2 - 4x + 4 - 32 - 4$$

$$g(x) = (x - 2)^2 - 36$$

The vertex of  $g(x) = (x + 4)(x - 8)$  is  $(2, -36)$ .

10. **(D)** Use the distance formula to find the missing coordinates of  $x$ .

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$\sqrt{(x_1 - 7)^2 + (4 - 5)^2} = 8$$

$$(x_1 - 7)^2 + 1 = 64$$

$$(x_1 - 7)^2 = 63$$

$$x - 7 = \pm\sqrt{63}$$

$$x = 7 \pm \sqrt{63}$$

$$x = 7 \pm 3\sqrt{7}$$

## Difficult Questions

For multiple-choice questions, solve the problem and pick the correct answer from the provided choices. Each multiple-choice question has only one correct answer. For student-produced response questions, solve each problem and write down your answers separately.

1. If  $n > 0$ , then  $n^2$  is
- (A) greater than  $n$ .
  - (B) equal to  $n$ .
  - (C) less than  $n$ .
  - (D) there is not enough information to solve this problem.

2. The half-life of an element is the amount of time that elapses until a certain mass of an element or compound decays to one-half its initial amount.

$$A = R_0 \left(\frac{1}{2}\right)^x$$

A certain compound's half-life can be approximated using the equation above where

$R_0$  = the initial amount of the compound

$A$  = the amount remaining after  $x$  years

$x$  = the time elapsed in years

If 125 grams of the compound is left to decay over 2.3 years, how many grams will remain?

- (A) 16.7  
(B) 25.4  
(C) 29.6  
(D) 41.1
3. Twenty-one students entered into a foul shooting contest in basketball. Each student was given five foul shots to attempt. The table below lists the number of foul shots made and the number of students who made that many foul shots.

Successful Foul Shots	Frequency
0	2
1	4
2	4
3	7
4	3
5	1

If a student who successfully shot at least 2 foul shots was chosen at random, what is the probability that the student successfully shot exactly 4 foul shots?

- (A)  $\frac{1}{7}$
- (B)  $\frac{1}{12}$
- (C)  $\frac{2}{15}$
- (D)  $\frac{3}{5}$

4. The number of students in an elementary class,  $s$ , decided to spend  $D$  dollars for a present for their favorite teacher, Mrs. Konopka. Each student was to contribute an equal amount. Because 15 students refused to contribute, each of those remaining agreed to contribute an additional, equal amount. What was the additional amount each had to contribute, in dollars?

- (A)  $(s - 15) \div D$
- (B)  $(s^2 - 15) \div D$
- (C)  $\frac{15D}{s^2 - 15s}$
- (D)  $\frac{s^2 - 15s}{15D}$

$$4a + n = 2a - 3$$

$$4b + m = 2b - 3$$

5. If  $n$  is  $m$  plus 1, which of the following is true?

- (A)  $a$  is  $b$  minus  $\frac{1}{2}$
- (B)  $a$  is  $b$  plus  $\frac{1}{2}$
- (C)  $a$  is  $b$  plus 1
- (D)  $a$  is  $b$  minus 2

6. Which of the following is equal to  $(\frac{m}{2} + n)^2$ ?

- (A)  $\frac{m^2}{2} + n^2$
- (B)  $\frac{m^2}{4} + mn + n^2$
- (C)  $\frac{m^2}{2} + mn + \frac{n^2}{4}$

(D)  $m + \frac{mn}{2} + n$

$$y = m(x + 3)(x - 5)$$

7. In the quadratic function above,  $m$  is a nonzero constant. The graph of the equation in the  $xy$ -plane has the vertex  $(e, f)$ . Which of the following is the value of  $f$ ?

- (A)  $16m$
- (B)  $4m$
- (C)  $-4m$
- (D)  $-16m$

$$y = x^2 + 5x + 8$$

$$y + 7x + 28 = 0$$

8. The system of equations above has how many solutions?

- (A) Exactly 3 solutions
- (B) Exactly 2 solutions
- (C) Exactly 1 solution
- (D) No solutions

9. If  $f(x) = 2x + 3$  and  $f(2x + 3) = 5x$ , what is the value of  $3x^2$ ?

$$f(x) = \frac{1}{(x - 3)^2 + 6(x - 3) + 9}$$

10. For what value of  $x$  is the above function  $f$  undefined?

## Answer Explanations

1. **(D)** Consider the three conditions that each satisfy  $n > 0$ :

- (a) Let  $n = 2$ , then  $n^2 = 4$

$$n^2 > n$$

(b) Let  $n = 1$ , then  $n^2 = 1$

$$n^2 = n$$

(c) Let  $n = \frac{1}{2}$ , then  $n^2 = \frac{1}{4}$

$$n^2 < n$$

Without more information about  $n$ , we cannot make a conclusion about  $n^2$ .

2. **(B)** Input 2.3 for the number of years,  $x$ , and input 125 for  $R_0$ , the initial amount of the compound.

$$\begin{aligned} A &= R_0 \left(\frac{1}{2}\right)^x \\ A &= 125 \left(\frac{1}{2}\right)^{2.3} \\ A &= 25.4 \end{aligned}$$

After 2.3 years, 125 grams of the compound will be reduced to 25.4 grams.

3. **(B)** Find the probability of selecting a student who successfully shot 4 foul shots by using the formula  $\frac{\text{favored outcomes}}{\text{all outcomes}}$ .

There are a total of 15 students who successfully shot 2 or more foul shots. Of those 15 students, 3 made exactly 4 baskets. Therefore,  $\frac{3}{15}$ , or  $\frac{1}{5}$ , of the students in the sample made exactly 4 baskets.

4. **(C)** If the entire class contributed equally, the cost per student, in terms of  $s$  and  $D$ , would be  $\frac{D}{s}$ . However, 15 of the students do not contribute this sum, so  $15\frac{D}{s}$  needs to be split among the remaining  $s - 15$  students.  $\frac{15\frac{D}{s}}{s - 15}$  can be rewritten as  $\left(15\frac{D}{s}\right)\left(\frac{1}{s - 15}\right)$ . By multiplying the numerators and denominators, we arrive at  $\frac{15D}{s^2 - 15s}$ .

5. **(A)** Subtract the two equations and combine like terms.

$$\begin{array}{r} 4a + n = 2a - 3 \\ - (4b + m = 2b - 3) \\ \hline 4a - 4b + n - m = 2a - 2b \\ 2a - 2b + n - m = 0 \end{array}$$

Substitute  $m + 1$  for  $n$ .

$$\begin{aligned} 2a - 2b + m + 1 - m &= 0 \\ 2a - 2b + 1 &= 0 \\ 2a - 2b &= -1 \\ a - b &= -\frac{1}{2} \\ a &= b - \frac{1}{2} \end{aligned}$$

6. **(B)** Use the model  $(a + b)^2 = a^2 + 2ab + b^2$  to expand the trinomial  $\left(\frac{m}{2} + n\right)^2$ .

$$\left(\frac{m}{2}\right)^2 + 2\left(\frac{m}{2}n\right) + (n)^2 = \frac{m^2}{4} + mn + n^2$$

The correct answer could also be found using the FOIL method of multiplication.

$$\begin{aligned} \left(\frac{m}{2} + n\right)^2 &= \left(\frac{m}{2} + n\right) + \left(\frac{m}{2} + n\right) \\ &= \frac{m^2}{4} + \frac{mn}{2} + \frac{mn}{2} + n^2 \\ &= \frac{m^2}{4} + mn + n^2 \end{aligned}$$

Choices (A), (C), and (D) are the results of improperly expanding the trinomial  $\left(\frac{m}{2} + n\right)^2$ .

7. **(D)** Use FOIL to expand the quantities in the parentheses.

$$y = m(x + 3)(x - 5) = m(x^2 - 2x - 15)$$

Multiply each term in the parentheses by  $m$ .

$$m(x^2 - 2x - 15) = mx^2 - 2xm - 15m$$

Find the value of  $e$  by using the formula  $e = -\frac{b}{2a}$ . (This is also the formula for finding the axis of symmetry of a parabola.)

$$e = -\frac{b}{2a} = -\frac{(-2m)}{2m} = 1$$

Find  $f$  by replacing  $x$  with 1 in  $mx^2 - 2xm - 15m$ .

$$m(1)^2 - 2(1)m - 15m = -16m$$

The value of  $f$  in the vertex is  $-16m$ .

8. **(C)** Set the second (linear) equation equal to  $y$ .

$$\begin{aligned}y + 7x + 28 &= 0 \\y &= -7x - 28\end{aligned}$$

Use the substitution principle to solve for  $x$ .

$$x^2 + 5x + 8 = -7x - 28$$

Set the equation equal to 0 and solve for  $x$ .

$$\begin{aligned}x^2 + 5x + 8 &= -7x - 28 \\x^2 + 12x + 36 &= 0 \\(x + 6)^2 &= 0 \\x + 6 &= 0\end{aligned}$$

Substitute  $x = -6$  into the equation  $y = x^2 + 5x + 8$  to find the value of  $y$ .

$$\begin{aligned}y &= (-6)^2 + 5(-6) + 8 \\y &= 14\end{aligned}$$

There is exactly one solution,  $(-6, 14)$ , to the system of equations.

9. **243** Use  $f(x) = 2x + 3$  and  $f(2x + 3)$  together to find the value of  $x$ . Since  $f(x) = 2x + 3$ , then  $f(2x + 3)$  must equal  $2(2x + 3) + 3$ . Set that expression equal to  $5x$  and solve for  $x$ .

$$2(2x + 3) + 3 = 5x$$

$$4x + 6 + 3 = 5x$$

$$9 = x$$

Substitute 9 for  $x$  in  $3x^2$ .

$$3(9^2) = 243$$

10. **0**

$$f(x) = \frac{1}{(x-3)^2 + 6(x-3) + 9}$$

Division by 0 is undefined so set the denominator of the function equal to 0.

$$(x-3)^2 + 6(x-3) + 9 = 0$$

$$x^2 - 6x + 9 + 6x - 18 + 9 = 0$$

$$x^2 = 0$$

$$x = 0$$

When  $x = 0$ , the function  $f$  is undefined.