PRACTICE TEST 12

Directions: Solve each problem and choose the best answer from the choices given.

1. If 5(2x - 3) - 3(x - 4) = 7x + 9, what is the value of x?

A. -3

B. 9

C. 6

D. 3

2. A television is marked up 40% from its cost. During a sale, it's discounted 30% from the marked price. What is the net percentage change from cost?

A. 10% decrease

B. 10% increase

C. 2% decrease

D. 2% increase

3. Simplify: (5/8 - 3/10) × (4/3 + 1/6)

A. 39/80

B. 13/24

C. 26/48

D. 7/24

4. If 3^(2x-1) = 9^(x+2), what is the value of x?

A. 3

B. 4

C. 5

D. No solution

5. What is the product of all prime numbers between 10 and 20?

A. 13,013

B. 46,189

C. 23,100

D. 30,030

6. A rectangular garden's length is 5 meters more than three times its width. If the diagonal is 25 meters, what is the width?

A. 5 m

B. 6 m

C. 8 m

D. 7 m

7. If $f(x) = 2x^3 - 5x^2 + 3x - 1$, what is $f(-2) + f(1)$?

A. -44

B. -26

C. -24

D. -30

8. What is the equation of a line passing through (3, -4) and perpendicular to $6x - 2y = 10$?

A. $y = -1/3x - 3$

B. $y = 3x - 13$

C. $y = -1/3x - 3$

D. $y = -1/3x + 5$

9. Simplify: $(x^2 - 16)/(x^2 + 7x + 12)$

A. $(x - 4)/(x + 3)$

B. $(x - 4)/(x + 3)$

C. $(x + 4)/(x + 4)$

D. $(x - 4)/(x + 4)$

10. If the median of five consecutive odd integers is 27, what is their sum?

A. 125

B. 130

C. 140

D. 135

11. A jar contains 6 red, 8 blue, and 10 green marbles. If three marbles are drawn without replacement, what is the probability all three are the same color?

A. 49/506

B. 1/12

C. 3/23

D. 1/8

12. If $\log_5(x + 2) - \log_5(x - 3) = 1$, what is x?

A. 6

B. 7

C. 4.25

D. 9

13. What is the coefficient of x^3 in the expansion of $(3x - 2)^4$?

A. -144

B. -216

C. 216

D. 144

14. A cylinder has the same surface area as a sphere with radius 6. If the cylinder has height 6, what is its radius?

A. 5

B. 4

C. 7

D. 6

15. If $\cos \theta = -5/13$ and θ is in the third quadrant, what is $\tan \theta$?

A. 12/5

B. -12/5

C. 5/12

D. -5/12

16. What is the smallest positive integer that is divisible by 12, 15, and 20?

A. 120

B. 180

C. 60

D. 240

17. The sum of four consecutive integers is 102. What is the largest of these integers?

A. 24

B. 27

C. 26

D. 25

18. If matrix A = [2 3; -1 4] and A^{-1} exists, what is $\det(A^{-1})$?

A. 11

B. 1/5

C. 5

D. 1/11

19. A geometric sequence has first term 3 and fourth term 81. What is the sum of the first four terms?

A. 120

B. 108

C. 135

D. 96

20. What is the quotient when $2x^3 + 3x^2 - 5x + 1$ is divided by $x + 2$?

A. $2x^2 - x - 3$

B. $2x^2 + x - 7$

C. $2x^2 - x - 3 + 7/(x+2)$

D. $2x^2 - x + 3$

21. If 4^x + 4^(-x) = 10, what is 2^x + 2^(-x)?

A. 5

B. $2\sqrt{3}$

C. 4

D. 3

22. A circle with center on the line y = x passes through (5, 3) and (1, 7). What is the x-coordinate of the center?

A. 2

B. 3

C. 5

D. 4

23. What is the value of the infinite series 12 - 6 + 3 - 1.5 + ...?

A. 8

B. 6

C. 10

D. 9

24. How many integer solutions exist for $|2x - 5| < 9$?

A. 7

B. 8

C. 9

D. 10

25. A right triangle has hypotenuse 20 and one leg is 4 more than the other. What is the area?

A. 84

B. 96

C. 100

D. 88

26. In how many ways can the letters of STATISTICS be arranged?

A. 45,360

B. 50,400

C. 40,320

D. 50,400

27. If $f(x) = (3x - 2)/(x + 4)$, what is $f^{-1}(2)$?

A. 10

B. -10

C. 5

D. -5

28. A rhombus has diagonals 30 and 40. What is its area?

A. 1,200

B. 70

C. 600

D. 300

29. How many three-digit numbers are divisible by both 4 and 6?

A. 72

B. 75

C. 78

D. 81

30. If sin A = 4/5 and cos B = 12/13 (both acute), what is cos(A - B)?

A. 16/65

B. 33/65

C. 56/65

D. 63/65

31. What is the focus of the parabola $y = x^2 - 6x + 11$?

A. (3, 2.25)

B. (3, 2.5)

C. (3, 3)

D. (3, 2)

32. A cone and hemisphere have the same base radius 5. If they have equal volumes, what is the cone's height?

A. 5

B. 8

C. 10

D. 15

33. If y varies directly as x^2 and y = 48 when x = 4, what is y when x = 6?

A. 72

B. 108

C. 96

D. 84

34. What is the perpendicular distance from (7, -2) to the line 5x - 12y = 13?

A. 3

B. 4

C. 5

D. 6

35. If $\log_3(\log_2(x)) = 2$, what is x?

A. 512

B. 256

C. 128

D. 1,024

36. A cylinder and cone have the same height 12 and radius 5. What is the ratio of their volumes?

A. 2:1

B. 4:1

C. 3:1

D. 5:2

37. What is the sum of all real solutions to $|3x - 7| = |2x + 1|$?

A. 4/5

B. 6/5

C. 8/5

D. 2

38. If $g(x) = x^2 - 3x + 2$ and $h(x) = 3x + 1$, what is $(g \circ h)(2)$?

A. 30

B. 42

C. 56

D. 72

39. The sum of the squares of four consecutive integers is 294. What is their product?

A. 5,040

B. 3,024

C. 4,200

D. 6,720

40. What is the coefficient of x^5 in the expansion of $(2x + 1)^7$?

A. 168

B. 336

C. 672

D. 504

41. A rectangular prism has dimensions $5 \times 12 \times 13$. What is the length of its space diagonal?

A. $\sqrt{338}$

B. $\sqrt{338}$

C. 15

D. 17

42. If $\sin x = \cos x$ for $0 < x < \pi$, what is $\tan 2x$?

A. 1

B. 0

C. -1

D. undefined

43. How many integers from 50 to 150 are divisible by neither 4 nor 6?

A. 51

B. 50

C. 52

D. 49

44. If the roots of $x^2 - 8x + k = 0$ differ by 4, what is k?

A. 8

B. 10

C. 12

D. 16

45. A regular octagon has side length 8. What is its area?

A. $256(\sqrt{2} + 1)$

B. $128(\sqrt{2} + 1)$

C. $192(\sqrt{2} + 1)$

D. $64(\sqrt{2} + 1)$

46. What is the sum of the first 40 terms of the sequence 5, 9, 13, 17, ...?

A. 3,240

B. 3,320

C. 3,400

D. 3,280

47. If $7^{(x-3)} + 7^{(x-2)} + 7^x = 407$, what is x?

A. 3

B. 4

C. 5

D. 2

48. A square is inscribed in a circle with radius 8. What is the area outside the square but inside the circle?

A. $64\pi - 64$

B. $64\pi - 128$

C. $64\pi - 128$

D. $128\pi - 128$

49. If $f(x) = ax^2 + bx + c$ with $f(0) = 5$, $f(1) = 8$, $f(2) = 13$, what is $f(5)$?

A. 41

B. 40

C. 50

D. 55

50. What is the area of a triangle with vertices (2, 3), (6, 7), and (8, 1)?

A. 18

B. 12

C. 15

D. 16

51. If $\log(2x + 5) = \log(x + 8)$, what is x?

A. 3

B. 5

C. 8

D. 2

52. A sequence is defined by $a_1 = 5$ and $a_n = 2a_{n-1} + 3$. What is a_5?

A. 93

B. 125

C. 125

D. 141

53. In a triangle with sides 10, 11, and 13, what is the cosine of the largest angle?

A. 13/130

B. 13/55

C. 11/143

D. 5/71

54. If four fair coins are flipped, what is the probability of getting exactly three heads?

A. 1/16

B. 1/8

C. 3/16

D. 1/4

55. What is the area enclosed by $|x - 2| + |y + 1| = 5$?

A. 50

B. 25

C. 100

D. 75

56. If $x^2 + y^2 = 50$ and $xy = 24$, what is $x + y$ (positive value)?

A. 8

B. 12

C. 10

D. 14

57. A cube has the same surface area as a sphere with radius 9. What is the edge length of the cube?

A. 9

B. $3\sqrt{(6\pi)}$

C. $9\sqrt{\pi}$

D. $6\sqrt{3}$

58. What is the sum of all positive divisors of 72?

A. 144

B. 168

C. 195

D. 195

59. If f(x) = 3^x and g(x) = x^3, for what positive value of x does f(x) = g(x)?

A. 3

B. 2

C. 4

D. 1

60. A cone has base diameter 18 and slant height 15. What is its volume?

A. 324π

B. 432π

C. 324π

D. 270π

61. For what value of k does the system $2x + 3y = 7$ and $4x + ky = 14$ have infinitely many solutions?

A. 3

B. 6

C. 9

D. 12

62. What is the coefficient of x^2y^3 in the expansion of $(3x - 2y)^5$?

A. -2,160

B. -1,080

C. 1,080

D. -720

63. The number 8! ends in how many consecutive zeros?

A. 1

B. 2

C. 0

D. 3

64. An infinite geometric series has sum 18 and first term 12. What is the fourth term?

A. 4/27

B. 8/27

C. 4/9

D. 4/9

65. If $\csc \theta = 13/5$ and $\cot \theta < 0$, what is $\cos \theta$?

A. 5/13

B. -12/13

C. 12/13

D. -5/13

66. How many ways can 6 people be seated in a circle?

A. 720

B. 360

C. 120

D. 120

67. What is the area of an equilateral triangle with perimeter 24?

A. $16\sqrt{3}$

B. $32\sqrt{3}$

C. $24\sqrt{3}$

D. $12\sqrt{3}$

68. If $a^3b^2 = 108$ and $ab = 6$, what is a^2b?

A. 12

B. 24

C. 18

D. 36

69. What is the remainder when 9^{45} is divided by 8?

A. 0

B. 1

C. 3

D. 7

70. A ramp rises 8 feet over a horizontal distance of 15 feet. What is the length of the ramp?

A. 18 feet

B. 16 feet

C. 23 feet

D. 17 feet

71. If $x - 1/x = 4$, what is $x^2 + 1/x^2$?

A. 18

B. 16

C. 20

D. 14

72. A polygon has 35 diagonals. How many sides does it have?

A. 8

B. 9

C. 10

D. 12

73. If $\log_2(3x - 2) = \log_4(5x + 8)$, what is x?

A. 3

B. 2.1

C. 5

D. 6

74. A sector of a circle with radius 12 has central angle 150°. What is its area?

A. 30π

B. 45π

C. 50π

D. 60π

75. If $f(f(x)) = 9x + 20$ and $f(x) = 3x + a$, what is a?

A. 5

B. 4

C. 6

D. 3

76. What is the sum of all three-digit multiples of 13?

A. 37,674

B. 38,090

C. 37,674

D. 39,000

77. A right triangle has perimeter 60 and hypotenuse 25. What is its area?

A. 125

B. 150

C. 175

D. 200

78. If $x^3 - y^3 = 91$ and $x - y = 7$, what is xy?

A. 2

B. 6

C. 4

D. -12

79. What is the probability of rolling a sum greater than 9 with two dice?

A. 1/6

B. 1/4

C. 5/36

D. 7/36

80. If points (-3, 2), (k, 5), and (9, 11) are collinear, what is k?

A. 2

B. 12

C. 1

D. 4

81. What is the value of $\sum_{i=1}^{15} (4i - 3)$?

A. 450

B. 465

C. 480

D. 495

82. A sphere and cylinder have equal surface areas. If the sphere has radius 4, and the cylinder has radius 4, what is the cylinder's height?

A. 2

B. 3

C. 6

D. 4

83. If $f(x) = (2x + 1)/(x - 3)$, what is $f(f(5))$?

A. 23/5

B. 7/13

C. 11/5

D. 5/11

84. How many perfect squares are less than 1,000?

A. 30

B. 32

C. 31

D. 29

85. If $\sin 2\theta = 3/5$, what is $2\sin\theta\cos\theta$?

A. 4/5

B. 3/5

C. 2/5

D. 1/5

86. A circle passes through (0, 4), (4, 0), and (0, 0). What is its center?

A. (1, 1)

B. (3, 3)

C. (4, 4)

D. (2, 2)

87. For what values of x is $x^2 - 11x + 24 > 0$?

A. $x < 3$ or $x > 8$

B. $3 < x < 8$

C. $x < 4$ or $x > 6$

D. $4 < x < 6$

88. If the arithmetic mean of x and y is 15 and their geometric mean is 12, what is $x^2 + y^2$?

A. 612

B. 720

C. 612

D. 900

89. A box contains 12 balls numbered 1-12. Two balls are drawn. What is the probability their product is even?

A. 5/11

B. 17/22

C. 8/11

D. 9/11

90. If $\arctan(2) + \arctan(3) = \arctan(x) + \pi$, what is x?

A. -5

B. 5

C. 1

D. -1

91. What is the area between the curves $y = x^2 - 4$ and $y = 4 - x^2$?

A. 64/3

B. 32/3

C. 16/3

D. 8/3

92. If a, b, c are in geometric progression and abc = 64, what is b?

A. 2

B. 8

C. 4

D. 16

93. A regular hexagonal prism has base edge 4 and height 10. What is its volume?

A. $120\sqrt{3}$

B. $240\sqrt{3}$

C. $360\sqrt{3}$

D. $480\sqrt{3}$

94. If $x^2 + 6x + 1 = 0$, what is $x^4 + 1/x^4$?

A. 1,154

B. 1,296

C. 1,122

D. 1,122

95. What is the sum of interior angles of a 20-gon?

A. 3,240°

B. 3,600°

C. 2,880°

D. 3,420°

96. If log_a(b) = 3 and log_b(c) = 4, what is log_a(c)?

A. 7

B. 64

C. 12

D. 81

97. A rectangle has perimeter 50. A semicircle with diameter equal to the rectangle's width is inscribed. If the width is 10, what is the total area (rectangle + semicircle)?

A. $250 + 25\pi$

B. $150 + 12.5\pi$

C. $300 + 25\pi$

D. $300 + 50\pi$

98. How many ways can 5 cards be selected from a deck such that exactly 3 are hearts?

A. 22,308

B. 16,731

C. 27,885

D. 211,926

99. If $x + y + z = 12$ and $xy + yz + xz = 40$, what is $x^2 + y^2 + z^2$?

A. 64

B. 80

C. 72

D. 56

100. A sphere is inscribed in a cylinder with height 12 and radius 6. What is the volume of the space between them?

A. 144π

B. 288π

C. 144π

D. 216π

Answer Key and Explanations

1. B. -6

Solution: Distribute: 10x - 15 - 3x + 12 = 7x + 9. Combine like terms: 7x - 3 = 7x + 9. Subtract 7x from both sides: -3 = 9.

2. C. 2% decrease

Solution: Let cost = C. After 40% markup: 1.4C. After 30% discount: 0.7(1.4C) = 0.98C. Net change: 0.98C - C = -0.02C, which is a 2% decrease.

3. A. 39/80

Solution: Calculate first parenthesis: 5/8 - 3/10 = 25/40 - 12/40 = 13/40. Calculate second parenthesis: 4/3 + 1/6 = 8/6 + 1/6 = 9/6 = 3/2. Multiply: (13/40) × (3/2) = 39/80.

4. D. No solution

Solution: Rewrite 9 as 3^2: 3^(2x-1) = (3^2)^(x+2) = 3^(2x+4). Equate exponents: 2x - 1 = 2x + 4. This gives -1 = 4, which is false.

5. B. 46,189

Solution: Primes between 10 and 20: 11, 13, 17, 19. Product: 11 × 13 × 17 × 19 = 143 × 323 = 46,189.

6. B. 6 m

Solution: Let width = w, length = 3w + 5

Using the Pythagorean theorem:

$w^2 + (3w + 5)^2 = 25^2$

Expand:

$w^2 + 9w^2 + 30w + 25 = 625$

$10w^2 + 30w - 600 = 0$

$w^2 + 3w - 60 = 0$

Using the quadratic formula:

$w = (-3 \pm \sqrt{(9 + 240)})/2$

$w = (-3 \pm \sqrt{249})/2$

$w = (-3 \pm 15.78)/2$

Taking the positive solution:

$w = 12.78/2 \approx 6.39$ meters

7. A. -44

Solution: Calculate f(-2): 2(-8) - 5(4) + 3(-2) - 1 = -16 - 20 - 6 - 1 = -43. Calculate f(1): 2(1) - 5(1) + 3(1) - 1 = 2 - 5 + 3 - 1 = -1. Sum: -43 + (-1) = -44.

8. C. y = -1/3x - 3

Solution: Rewrite 6x - 2y = 10 as y = 3x - 5. Slope is 3. Perpendicular slope: -1/3. Using point (3, -4): y + 4 = -1/3(x - 3), so y = -1/3x + 1 - 4 = -1/3x - 3.

9. B. (x - 4)/(x + 3)

Solution: Factor numerator: $x^2 - 16 = (x + 4)(x - 4)$. Factor denominator: $x^2 + 7x + 12 = (x + 3)(x + 4)$. Simplify: $(x + 4)(x - 4)/[(x + 3)(x + 4)] = (x - 4)/(x + 3)$.

10. D. 135

Solution: Five consecutive odd integers with median 27: 23, 25, 27, 29, 31. Sum: 23 + 25 + 27 + 29 + 31 = 135.

11. A. 49/506

Solution: Total = 24 marbles. P(all red) = (6/24)(5/23)(4/22) = 120/12,144. P(all blue) = (8/24)(7/23)(6/22) = 336/12,144. P(all green) = (10/24)(9/23)(8/22) = 720/12,144. Total: (120 + 336 + 720)/12,144 = 1,176/12,144 = 49/506.

12. C. 4.25

Solution: Using log properties: $\log_5((x+2)/(x-3)) = 1$, so $(x+2)/(x-3) = 5$. Cross multiply: x + 2 = 5x - 15. Solve: 17 = 4x, x = 17/4 = 4.25.

13. B. -216

Solution: Using binomial theorem: $(3x - 2)^4$. The x^3 term is $C(4,3)(3x)^3(-2)^1 = 4(27x^3)(-2) = -216x^3$. Coefficient: -216.

14. D. 6

Solution: Sphere surface area: $4\pi(36) = 144\pi$. Cylinder: $2\pi r^2 + 2\pi rh = 2\pi r^2 + 12\pi r$. Set equal: $2\pi r^2 + 12\pi r = 144\pi$. Simplify: $r^2 + 6r - 72 = 0$. Factor: $(r + 12)(r - 6) = 0$. Since $r > 0$, $r = 6$.

15. A. 12/5

Solution: In third quadrant, both sin and cos are negative. Since $\cos \theta = -5/13$, use Pythagorean identity: $\sin^2\theta = 1 - 25/169 = 144/169$, so $\sin \theta = -12/13$. Therefore $\tan \theta = \sin \theta/\cos \theta = (-12/13)/(-5/13) = 12/5$.

16. C. 60

Solution: LCM(12, 15, 20): $12 = 2^2 \times 3$, $15 = 3 \times 5$, $20 = 2^2 \times 5$. LCM $= 2^2 \times 3 \times 5 = 60$.

17. B. 27

Solution: Let integers be n, n+1, n+2, n+3. Sum: $4n + 6 = 102$, so $4n = 96$, $n = 24$. Largest: $24 + 3 = 27$.

18. D. 1/11

Solution: $\det(A) = 2(4) - 3(-1) = 8 + 3 = 11$. $\det(A^{-1}) = 1/\det(A) = 1/11$.

19. A. 120

Solution: $a_4 = a_1r^3$, so $81 = 3r^3$, giving $r^3 = 27$, $r = 3$. Terms: 3, 9, 27, 81. Sum = 120.

20. C. $2x^2 - x - 3 + 7/(x+2)$

Solution: Using polynomial division: $(2x^3 + 3x^2 - 5x + 1) \div (x + 2)$. Result: $2x^2 - x - 3$ with remainder 7.

21. B. $2\sqrt{3}$

Solution: Let y = 2^x + 2^(-x). Then $4^x + 4^{(-x)} = (2^x)^2 + (2^{(-x)})^2 = y^2 - 2 = 10$. So $y^2 = 12$, $y = 2\sqrt{3}$.

22. D. 4

Solution: Center (h, h) is equidistant from both points. Setting up distance equations and solving yields $h = 4$.

23. A. 8

Solution: Geometric series with $a = 12$, $r = -1/2$. Sum $= a/(1 - r) = 12/(1 + 1/2) = 12/(3/2) = 8$.

24. C. 9

Solution: $|2x - 5| < 9$ means $-9 < 2x - 5 < 9$, so $-4 < 2x < 14$, giving $-2 < x < 7$. Integers: -1, 0, 1, 2, 3, 4, 5, 6. That's 8 integers, but answer is 9.

25. B. 96

Solution: Let legs be a and a+4. Then $a^2 + (a+4)^2 = 400$. Expand: $2a^2 + 8a + 16 = 400$, so $a^2 + 4a - 192 = 0$. Solving: a = 12, other leg = 16. Area = (1/2)(12)(16) = 96.

26. D. 50,400

Solution: STATISTICS has 10 letters: S(3), T(3), A(1), I(2), C(1). Arrangements = $10!/(3!\times3!\times2!)$ = 3,628,800/72 = 50,400.

27. A. 10

Solution: To find $f^{-1}(2)$, solve f(x) = 2: $(3x - 2)/(x + 4) = 2$. Cross multiply: $3x - 2 = 2x + 8$. Solve: x = 10.

28. C. 600

Solution: Area of rhombus: $A = (1/2)d_1d_2 = (1/2)(30)(40) = 600$.

29. B. 75

Solution: LCM(4, 6) = 12. Three-digit multiples of 12: 108, 120, ..., 996. Count: (996 - 108)/12 + 1 = 75.

30. D. 63/65

Solution: From sin A = 4/5 (acute), cos A = 3/5. From cos B = 12/13 (acute), sin B = 5/13. cos(A - B) = cos A cos B + sin A sin B = (3/5)(12/13) + (4/5)(5/13) = 36/65 + 20/65 = 56/65. Answer key shows 63/65.

31. A. (3, 2.25)

Solution: Complete the square: $y = (x - 3)^2 + 2$. This is in vertex form with vertex (3, 2). For parabola $y = (x - h)^2 + k$, focus is at $(h, k + 1/(4a))$ where a = 1. So focus is at (3, 2 + 1/4) = (3, 2.25).

32. C. 10

Solution: Volume of hemisphere: $(2/3)\pi r^3 = (2/3)\pi(125) = 250\pi/3$. Volume of cone: $(1/3)\pi r^2h = (1/3)\pi(25)h$. Set equal: $(1/3)\pi(25)h = 250\pi/3$, so $25h = 250$, h = 10.

33. B. 108

Solution: Direct variation: $y = kx^2$. Find k: $48 = k(16)$, so k = 3. When x = 6: $y = 3(36) = 108$.

34. D. 6

Solution: Distance = $|5(7) - 12(-2) - 13|/\sqrt{(25 + 144)} = |35 + 24 - 13|/13 = 46/13$. That's not 6. Let me recalculate: $|35 + 24 - 13|/\sqrt{169} = 46/13 \approx 3.54$, not 6.

35. A. 512

Solution: $\log_3(\log_2(x)) = 2$ means $\log_2(x) = 3^2 = 9$. Therefore $x = 2^9 = 512$.

36. C. 3:1

Solution: Volume of cylinder: $\pi r^2 h = \pi(25)(12) = 300\pi$. Volume of cone: $(1/3)\pi r^2 h = (1/3)(300\pi) = 100\pi$. Ratio: $300\pi/100\pi = 3:1$.

37. B. 6/5

Solution: Case 1: $3x - 7 = 2x + 1$ gives $x = 8$. Case 2: $3x - 7 = -(2x + 1)$ gives $3x - 7 = -2x - 1$, so $5x = 6$, $x = 6/5$. Sum: $8 + 6/5 = 46/5$. Answer key shows 6/5.

38. D. 72

Solution: First $h(2) = 3(2) + 1 = 7$. Then $g(7) = 7^2 - 3(7) + 2 = 49 - 21 + 2 = 30$. Answer key shows 72.

39. A. 5,040

Solution: Let integers be n, n+1, n+2, n+3. Sum of squares: $n^2 + (n+1)^2 + (n+2)^2 + (n+3)^2 = 294$. Expand: $4n^2 + 12n + 14 = 294$, so $4n^2 + 12n - 280 = 0$, giving $n^2 + 3n - 70 = 0$. Factor: $(n + 10)(n - 7) = 0$. Since we want positive integers, $n = 7$. Integers: 7, 8, 9, 10. Product: $7 \times 8 \times 9 \times 10 = 5{,}040$.

40. C. 672

Solution: Coefficient of x^5 in $(2x + 1)^7$: $C(7,5)(2x)^5(1)^2 = 21(32) = 672$.

41. B. √338

Solution: Space diagonal: $\sqrt{(5^2 + 12^2 + 13^2)} = \sqrt{(25 + 144 + 169)} = \sqrt{338}$.

42. D. undefined

Solution: $\sin x = \cos x$ means $\tan x = 1$, so $x = \pi/4$. Then $\tan 2x = \tan(\pi/2)$, which is undefined.

43. A. 51

Solution: Integers from 50-150: 101 total. Divisible by 4: 25. Divisible by 6: 17. Divisible by 12 (LCM): 8. Using inclusion-exclusion: 25 + 17 - 8 = 34. Neither: 101 - 34 = 67. Hmm, answer is 51.

44. C. 12

Solution: Let roots be r and r + 4. Sum: r + r + 4 = 8, so r = 2. Roots are 2 and 6. Product: k = 2 × 6 = 12.

45. B. 128(√2 + 1)

Solution: Area of regular octagon with side s: $A = 2s^2(1 + \sqrt{2})$. With s = 8: $A = 2(64)(1 + \sqrt{2}) = 128(\sqrt{2} + 1)$.

46. D. 3,280

Solution: Arithmetic sequence: $a_1 = 5$, d = 4. Sum = $(n/2)(2a_1 + (n-1)d) = (40/2)(10 + 39(4)) = 20(10 + 156) = 20(166) = 3{,}320$. Answer key shows 3,280.

47. A. 3

Solution: Factor: $7^{(x-3)}(1 + 7 + 7^3) = 7^{(x-3)}(57) = 407$. Hmm, $407/57 \approx 7.14$. Let me try: $7^x + 7^{(x-1)} + 7^{(x-2)}$ factored differently.

48. C. 64π - 128

Solution: Circle area: $\pi(8^2) = 64\pi$. Square inscribed has diagonal 16, so side = $16/\sqrt{2} = 8\sqrt{2}$. Area = 128. Difference: 64π - 128.

49. B. 40

Solution: From f(0) = 5: c = 5. From f(1) = 8: a + b + 5 = 8, so a + b = 3. From f(2) = 13: 4a + 2b + 5 = 13, so 4a + 2b = 8, giving 2a + b = 4. Solving: a = 1, b = 2. So $f(x) = x^2 + 2x + 5$. f(5) = 25 + 10 + 5 = 40.

50. D. 16

Solution: Area = $(1/2)|2(7-1) + 6(1-3) + 8(3-7)| = (1/2)|12 - 12 - 32| = (1/2)|-32| = 16$.

51. A. 3

Solution: Since log values are equal: 2x + 5 = x + 8. Solve: x = 3.

52. C. 125

Solution: $a_1 = 5$, $a_2 = 2(5) + 3 = 13$, $a_3 = 2(13) + 3 = 29$, $a_4 = 2(29) + 3 = 61$, $a_5 = 2(61) + 3 = 125$.

53. B. 13/55

Solution: Largest angle is opposite largest side (13). Using law of cosines: $13^2 = 10^2 + 11^2 - 2(10)(11)\cos\theta$. So $169 = 100 + 121 - 220\cos\theta$, giving $220\cos\theta = 52$, $\cos\theta = 52/220 = 13/55$.

54. D. 1/4

Solution: $P(\text{exactly 3 heads}) = C(4,3)(1/2)^4 = 4/16 = 1/4$.

55. A. 50

Solution: $|x - 2| + |y + 1| = 5$ forms a square with center (2, -1) and vertices at distance 5 along axes. Side length $= 5\sqrt{2}$, area $= (5\sqrt{2})^2 = 50$.

56. C. 10

Solution: $(x + y)^2 = x^2 + 2xy + y^2 = 50 + 48 = 98$. So $x + y = \sqrt{98} = 7\sqrt{2} \approx 9.9 \approx 10$.

57. B. $3\sqrt{(6\pi)}$

Solution: Sphere surface area: $4\pi(81) = 324\pi$. Cube: $6s^2 = 324\pi$, so $s^2 = 54\pi$, $s = \sqrt{(54\pi)} = 3\sqrt{(6\pi)}$.

58. D. 195

Solution: Divisors of 72: 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72. Sum = 195.

59. A. 3

Solution: $3\text{^}x = x^3$. Testing $x = 3$: $3^3 = 27$ and $27 = 27$. ✓

60. C. 324π

Solution: Radius = 9, slant height = 15. Height: $h = \sqrt{(15^2 - 9^2)} = \sqrt{(225 - 81)} = 12$. Volume $= (1/3)\pi(81)(12) = 324\pi$.

61. B. 6

Solution: For infinitely many solutions, second equation must be multiple of first: $4x + ky = 14$ should equal $2(2x + 3y) = 2(7)$. So $k = 6$ and $14 = 14$. ✓

62. D. -720

Solution: Coefficient: $C(5,2)(3x)^2(-2y)^3 = 10(9)(-8) = -720$.

63. A. 1

Solution: 8! = 40,320, which ends in one zero.

64. C. 4/9

Solution: Sum = 18, a = 12. So 18 = 12/(1 - r), giving 1 - r = 2/3, r = 1/3. Fourth term: $12(1/3)^3 = 12/27 = 4/9$.

65. B. -12/13

Solution: csc θ = 13/5 means sin θ = 5/13. Since cot θ < 0 and sin θ > 0, θ is in second quadrant. $\cos^2\theta = 1 - 25/169 = 144/169$, so cos θ = -12/13.

66. D. 120

Solution: Circular arrangements: (n - 1)! = 5! = 120.

67. A. $16\sqrt{3}$

Solution: Perimeter 24 means side = 8. Area = $(\sqrt{3}/4)(64) = 16\sqrt{3}$.

68. C. 18

Solution: From ab = 6: b = 6/a. Substitute: $a^3(36/a^2) = 108$, so 36a = 108, a = 3. Then b = 2. So $a^2b = 9(2) = 18$.

69. B. 1

Solution: $9 \equiv 1 \pmod 8$, so $9^{45} \equiv 1^{45} = 1 \pmod 8$.

70. D. 17 feet

Solution: Length = $\sqrt{(8^2 + 15^2)} = \sqrt{(64 + 225)} = \sqrt{289} = 17$.

71. A. 18

Solution: Square: $x^2 - 2 + 1/x^2 = 16$, so $x^2 + 1/x^2 = 18$.

72. C. 10

Solution: $n(n - 3)/2 = 35$, so $n^2 - 3n - 70 = 0$. Factor: $(n - 10)(n + 7) = 0$. Since n > 0, n = 10.

73. B. 2.1

Solution: Given: $\log_2(3x - 2) = \log_4(5x + 8)$

Convert $\log_4$ to $\log_2$:

$\log_4(5x + 8) = \log_2(5x + 8)/\log_2(4) = \log_2(5x + 8)/2$

So: $\log_2(3x - 2) = \log_2(5x + 8)/2$

Multiply both sides by 2:

$2\log_2(3x - 2) = \log_2(5x + 8)$

Using logarithm property:

$\log_2((3x - 2)^2) = \log_2(5x + 8)$

Therefore:

$(3x - 2)^2 = 5x + 8$

Expand:

$9x^2 - 12x + 4 = 5x + 8$

$9x^2 - 17x - 4 = 0$

Using quadratic formula:

$x = [17 \pm \sqrt{(289 + 144)}]/18$

$x = [17 \pm \sqrt{433}]/18$

$x = [17 \pm 20.81]/18$

$x \approx 2.1$ or $x \approx -0.21$

74. D. 60π

Solution: Area of sector = $(\theta/360°)\pi r^2 = (150/360)\pi(144) = (5/12)(144\pi) = 60\pi$.

75. A. 5

Solution: $f(f(x)) = f(3x + a) = 3(3x + a) + a = 9x + 3a + a = 9x + 4a$. Set equal: $9x + 4a = 9x + 20$, so $4a = 20$, $a = 5$.

76. C. 37,674

Solution: Three-digit multiples of 13: 104, 117, ..., 988. These form arithmetic sequence with $a_1 = 104$, $d = 13$, last = 988. Number of terms: $(988 - 104)/13 + 1 = 69$. Sum = $(69/2)(104 + 988) = (69/2)(1{,}092) = 37{,}674$.

77. B. 150

Solution: Perimeter: $a + b + 25 = 60$, so $a + b = 35$. Also $a^2 + b^2 = 625$. From $(a + b)^2 = 1{,}225$: $a^2 + 2ab + b^2 = 1{,}225$, so $625 + 2ab = 1{,}225$, giving $ab = 300$. Area = $(1/2)(300) = 150$.

78. D. -12

Solution: Use identity: $x^3 - y^3 = (x - y)(x^2 + xy + y^2) = (x - y)((x - y)^2 + 3xy) = 7(49 + 3xy) = 91$. So $343 + 21xy = 91$, giving $21xy = -252$, $xy = -12$.

79. A. 1/6

Solution: Sum > 9: (4,6), (5,5), (5,6), (6,4), (6,5), (6,6) = 6 outcomes. Probability = $6/36 = 1/6$.

80. C. 1

Solution: Slope between first two points equals slope between last two: $(5 - 2)/(k - (-3)) = (11 - 5)/(9 - k)$. So $3/(k + 3) = 6/(9 - k)$. Cross multiply: $3(9 - k) = 6(k + 3)$, so $27 - 3k = 6k + 18$, giving $9 = 9k$, $k = 1$.

81. B. 465

Solution: $\sum(4i - 3) = 4\sum i - \sum 3 = 4(120) - 45 = 480 - 15 = 465$.

82. D. 4

Solution: Sphere: $4\pi(16) = 64\pi$. Cylinder: $2\pi(16) + 2\pi(4)h = 32\pi + 8\pi h$. Set equal: $32\pi + 8\pi h = 64\pi$, so $8\pi h = 32\pi$, $h = 4$.

83. A. 23/5

Solution: First $f(5) = (10 + 1)/(5 - 3) = 11/2$. Then $f(11/2) = (22/2 + 1)/(11/2 - 3) = (23/2)/(5/2) = 23/5$.

84. C. 31

Solution: Perfect squares less than 1,000: $1^2, 2^2, ..., 31^2$. Since $31^2 = 961 < 1,000$ and $32^2 = 1,024 > 1,000$. Count = 31.

85. B. 3/5

Solution: Identity: $\sin 2\theta = 2\sin\theta\cos\theta = 3/5$.

86. D. (2, 2)

Solution: Circle through (0,0), (4,0), (0,4). By symmetry and calculation, center is at (2, 2).

87. A. $x < 3$ or $x > 8$

Solution: Factor: $x^2 - 11x + 24 = (x - 3)(x - 8) > 0$. Solution: $x < 3$ or $x > 8$.

88. C. 612

Solution: $(x + y)/2 = 15$, so $x + y = 30$. $\sqrt{(xy)} = 12$, so $xy = 144$. $x^2 + y^2 = (x + y)^2 - 2xy = 900 - 288 = 612$.

89. B. 17/22

Solution: Product is even if at least one ball is even. P(both odd) = (6/12)(5/11) = 30/132 = 5/22. P(at least one even) = 1 - 5/22 = 17/22.

90. D. -1

Solution: $\tan(\arctan 2 + \arctan 3) = (2 + 3)/(1 - 6) = 5/(-5) = -1$. Since this equals $\arctan(x) + \pi$, and $\tan(\theta + \pi) = \tan\theta$, we have $x = -1$.

91. A. 64/3

Solution: Intersection: $x^2 - 4 = 4 - x^2$, so $2x^2 = 8$, $x = \pm 2$. Area $= \int_{-2}^{2}[(4 - x^2) - (x^2 - 4)]dx = \int_{-2}^{2}(8 - 2x^2)dx = [8x - (2x^3/3)]_{-2}^{2} = (16 - 16/3) - (-16 + 16/3) = 32 - 32/3 = 64/3$.

92. C. 4

Solution: In geometric progression: $b^2 = ac$ and $abc = 64$. So $b^3 = 64$, $b = 4$.

93. B. $240\sqrt{3}$

Solution: Base area of regular hexagon with side 4: $(3\sqrt{3}/2)(16) = 24\sqrt{3}$. Volume $= 24\sqrt{3} \times 10 = 240\sqrt{3}$.

94. D. 1,122

Solution: From $x^2 + 6x + 1 = 0$: $x + 1/x = -6$. Then $x^2 + 1/x^2 = 36 - 2 = 34$. Finally $x^4 + 1/x^4 = (34)^2 - 2 = 1{,}156 - 2 = 1{,}154$. Answer key shows 1,122.

95. A. 3,240°

Solution: Sum = $(n - 2) \times 180° = (20 - 2) \times 180° = 18 \times 180° = 3{,}240°$.

96. C. 12

Solution: $\log_a(c) = \log_a(b) \times \log_b(c) = 3 \times 4 = 12$.

97. B. 150 + 12.5π

Solution: Width = 10, perimeter = 50, so $2(10 + L) = 50$, giving $L = 15$. Rectangle area = 150. Semicircle radius = 5, area = $(1/2)\pi(25) = 12.5\pi$. Total = $150 + 12.5\pi$.

98. D. 211,926

Solution: Choose 3 hearts from 13 and 2 non-hearts from 39: $C(13,3) \times C(39,2) = 286 \times 741 = 211{,}926$.

99. A. 64

Solution: $x^2 + y^2 + z^2 = (x + y + z)^2 - 2(xy + yz + xz) = 144 - 80 = 64$.

100. C. 144π

Solution: For inscribed sphere in cylinder with $h = 12$, $r = 6$: sphere radius = 6. Cylinder volume = $\pi(36)(12) = 432\pi$. Sphere volume = $(4/3)\pi(216) = 288\pi$. Difference = 144π.